

EELE 477
Digital Signal Processing

5b

Implementing FIR Systems

FIR Computation

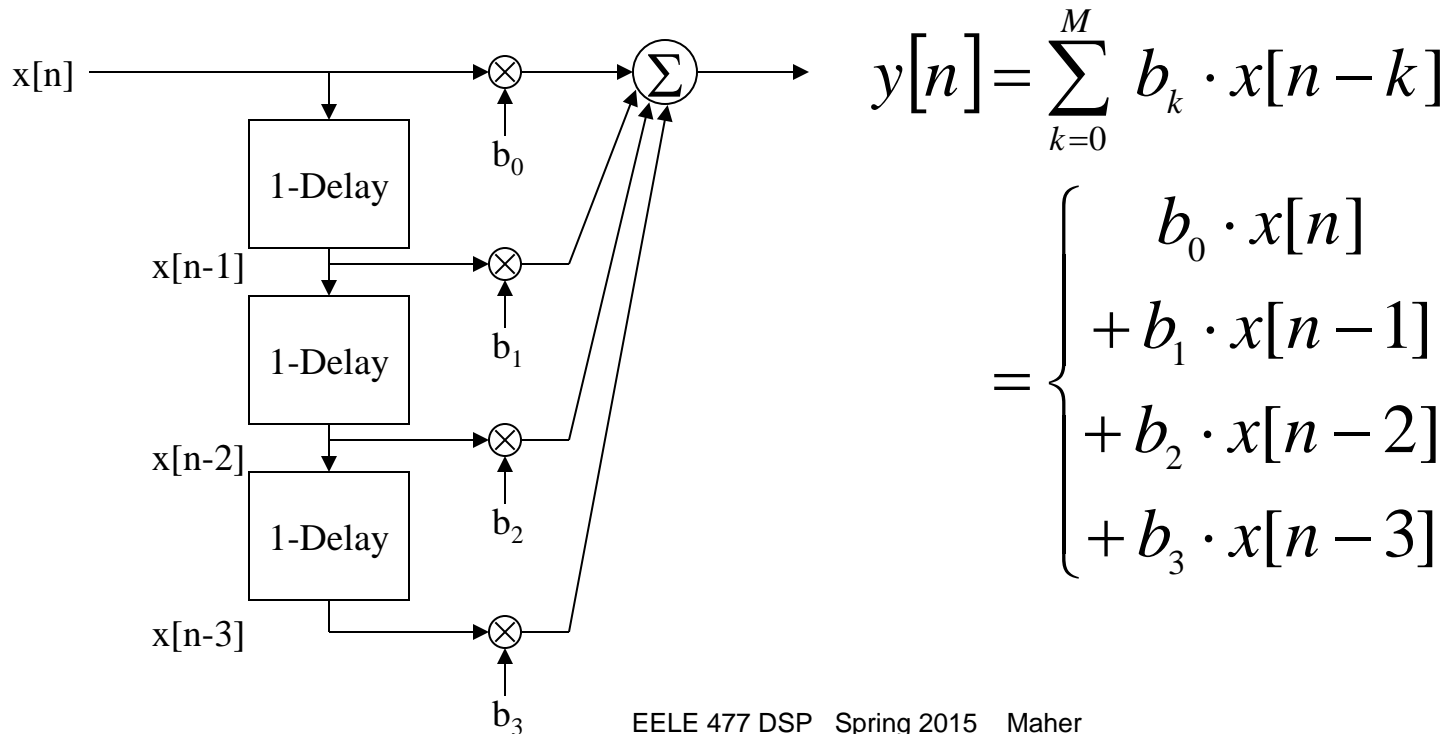
- The general definition of FIR:

$$y[n] = \sum_{k=0}^M b_k \cdot x[n-k]$$

- Requires
 - Delayed values of input $x[n]$
 - Multiply coefficients b_k
 - Sum up partial products

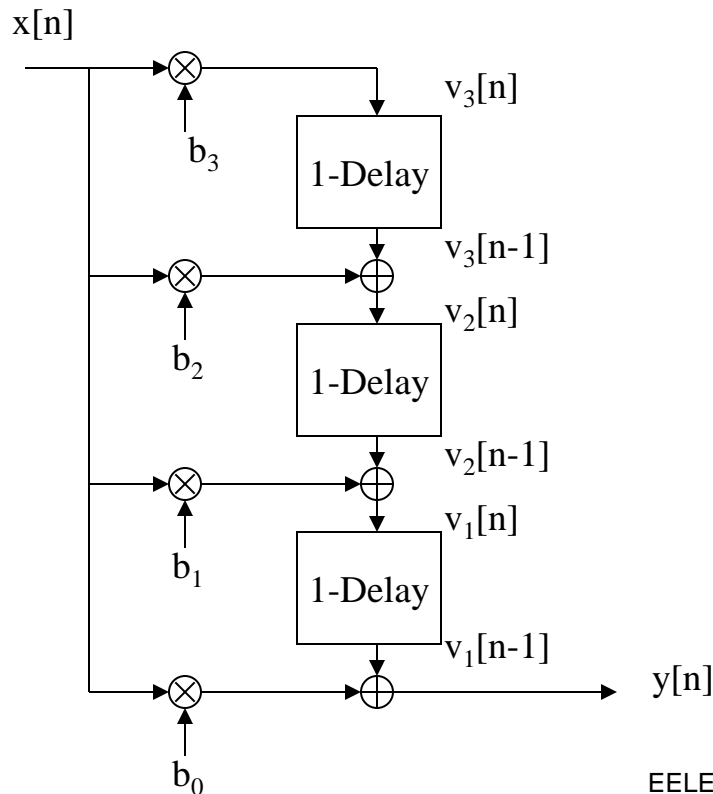
FIR Signal Flow Diagram

- Use *unit delay* elements, multipliers, and sum:



Another Flow Diagram

- Analyze to show the same result:



$$\begin{aligned}
 y[n] &= b_0 \cdot x[n] + v_1[n-1] \\
 &= b_0 \cdot x[n] + b_1 \cdot x[n-1] + v_2[n-2] \\
 &= b_0 \cdot x[n] + b_1 \cdot x[n-1] + b_2 \cdot x[n-2] + v_3[n-3] \\
 &= b_0 \cdot x[n] + b_1 \cdot x[n-1] + b_2 \cdot x[n-2] + b_3 \cdot x[n-3]
 \end{aligned}$$

Time Invariant

- A time invariant system: delaying the input simply delays the output.
- If $f()$ is LTI, then:

$$\text{if } y[n] = f(x[n]), \rightarrow y[n - n_0] = f(x[n - n_0])$$

Linearity

- A linear system: scaling and summing various inputs simply scales and sums the corresponding outputs.
- Linearity implies *superposition*:

$$\text{if } y_1[n] = f(x_1[n]) \text{ and } y_2[n] = f(x_2[n]), \text{ then}$$
$$f(\alpha \cdot x_1[n] + \beta \cdot x_2[n]) = \alpha \cdot y_1[n] + \beta \cdot y_2[n]$$

LTI: Linear Time Invariant

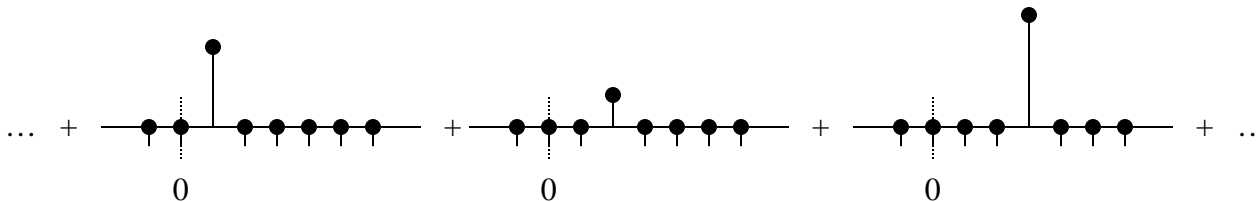
- LTI systems are an important class of systems
- Not *all* useful systems are LTI
- If we know a system is LTI, then we know that it can be fully described by its *unit sample response*, since we can represent the output as a delayed and scaled sum (time shift and superposition).

Convolution and $\delta[n]$

- Recall that:

$$x[n] = \sum_{\text{all } l} x[l] \cdot \delta[n - l]$$

Expresses $x[n]$ as a sum of shifted and scaled impulses:



Convolution (cont.)

- So, applying the shifted and scaled impulses to an LTI system means the output is a set of shifted and scaled impulse responses!

$$y[n] = \sum_{\text{all } l} x[l] \cdot h[n-l]$$

- All LTI systems can be represented this way.

Convolution (cont.)

- Convolution operation is associative, commutative, and distributive
- Cascaded (sequential) LTI systems imply convolution sequence:

