CHAPTER 3

Basic Physical Quantities and Laws

Musicians, scientists, and audiophiles each have their own specialized terminology. Once you have sufficient exposure to the terminology of a particular discipline and understand it, you are able to use it in meaningful ways. Our study of music, speech, and audio as it applies to acoustics requires some familiarity with the terminology and concepts involved. In this chapter we consider some fundamental physical quantities and laws that form the foundation upon which we build our physical description of musical instruments, the human speech mechanism, and audio systems.

3.1 Four Fundamental Physical Quantities

All the terms used in science must be defined carefully to avoid possible confusion and misunderstanding. As you will see, once a term has been defined, it is used to define still other terms, and in this manner the scientific jargon is formulated. However, there must be a starting point; that is, several basic quantities must be used to start the hierarchy of definitions. For the physics which we will use, four fundamental quantities are needed. The choice of which physical quantities are taken as fundamental is somewhat arbitrary, but the three usually selected are length, time, and mass. Even though electric current is now frequently chosen as the fourth fundamental physical quantity, we will choose electric charge for reasons of conceptual clarity. These quantities are used to define all the other physical quantities which will follow, so they can only be defined through measurement; that is, we cannot define length independent of measurement since length is "defined" by the act of making a measurement. Likewise, time, mass, and charge have meaning in physics only in terms of their measurement.

Length (L) is the first fundamental quantity. It can be defined as the spatial distance between two points. The measurement is accomplished by comparing the unknown distance to some standard length such as the **standard meter**. Historically, the meter was defined to be one ten-millionth the distance from the North Pole to the Equator. Later, for practical reasons, the standard meter (m) was defined as the distance between two lines engraved on gold plugs near the ends of a specially made platinum-iridium bar stored at the International Bureau of Weights and Measures near Paris. This rather intuitive definition of the meter will form the basis for our concept of length. Length, then, is defined by comparing an unknown distance to a measuring device, such as a meter stick. Convenient units of length for many acoustical measurements are the centimeter and the millimeter, defined respectively as one one-hundredth and one one-thousandth of a meter. (To further refine the accuracy of length measurements, the standard meter has been redefined recently as the distance light travels through vacuum in 1/299,792,458 of a second.)

Time (t) is the second fundamental quantity to be considered. As with length, we have an intuitive feeling for time even though we would probably have great difficulty in defining it. We may think of time in terms of the duration of events, and of the measurement of time as the comparing of the duration of an unknown event with that of a standard event. We define a solar day as the time it takes the Earth to make one complete rotation on its axis, so we have a standard event which can be used to measure time. Careful measurement, however, has shown that the solar day varies slightly in time during the course of the year; so a mean solar day was defined. This is just the "time length" of a solar day averaged over a year. The mean solar day can then be subdivided into 24 hours, each hour into 60 minutes, and each minute into 60 seconds. Thus there are 24 ? 60 ? 60 = 86,400 seconds in a mean solar day. The **standard second** (s) can be defined as 1/86,400 of a mean solar day. Recent astronomical measurements have shown that the rotating Earth can no longer be regarded as a satisfactory clock because there are slight irregularities in its rotation. Just as the meter was redefined to improve the accuracy of length measurements, the standard second was redefined as the time required for 9,192,631,770 vibrations of a certain wavelength of a cesium isotope.)

Mass (m) is the third fundamental quantity to be considered. The concept of mass is not as intuitively obvious as length and time, but we can try to develop some feeling for this concept through the following experiment. Suppose you are walking barefoot along a country lane and you see an old tin can. You kick the can as hard as possible and it sails down the street. Farther down the lane you encounter a similar tin can, which you also kick. This time, however, someone has previously filled the can with lead. When you kick the can it barely moves, and you experience considerable pain in your foot. One way to describe the difference between the two cans is to say that the second can (because of all the lead) has a greater mass than the first can. The mass of an object is related to the difficulty of changing the state of motion of the object. To determine the mass of any object it is only necessary to compare the unknown to a standard mass by use of a balance. The international standard of mass is the **standard kilogram** (kg), standardized as the mass of a particular platinum-iridium cylinder kept with the standard meter at the International Bureau of Weights and Measures. The gram, defined as one one-thousandth of a kilogram, is often used to express mass.

All objects are composed of particles possessing **electric charge** (q), a fourth fundamental quantity. Friction can "rub off' some of these particles so that the effect of isolated charges can be demonstrated, even if the charge itself is not directly observable. For example, an ordinary hard rubber comb, when drawn through your hair, becomes charged and will attract small pieces of paper. Experiments show that when two materials are charged by being rubbed together, the resulting charge of each material is fundamentally different from the other. To differentiate the two types of charge, one is labeled negative, as is the charge found on the rubber comb, while the other is called positive. (These labels are arbitrary, albeit conventional. The charges could just as easily have been called red and green.) It is now known that electrons all possess an identical negative charge; hence, a negatively charged comb is a comb with an excess of electrons. Like the other fundamental quantities, charge is defined operationally by comparison to a standard. A **coulomb** (C) of electric charge is defined as the total charge of $6.24 ? 10^{18}$ electrons, that is, about six and a quarter billion billion electrons.

3.2 Derived Quantities

The following quantities are defined in terms of the four fundamental quantities. The dimensions associated with each quantity identify the manner in which it was obtained from the fundamental quantities.

When an object changes position, it is said to undergo a **displacement.** The magnitude of that displacement is the length or distance it moved. Distance alone, however, is not sufficient to determine displacement. Displacement must also include the direction in which the object is moved. In many practical applications displacement is measured from some natural or rest position, as, for example, when considering the displacement of a point on a guitar string. To completely specify the displacement, we must specify not only the distance the string has moved but also the direction of motion from the string's rest position.

If we enclose a rectangular region on a flat tabletop by drawing two pairs of parallel, but mutually perpendicular, lines we can define the **surface area** (S) of the enclosed region as the product of the length of two of the perpendicular sides. The unit used to measure area is the square meter or something comparable. (Area can be defined for nonrectangular regions on a flat surface, but the definition becomes more complicated.) If we enclose a "rectangular" space with three sets of mutually perpendicular walls, we can define the **volume** (V) of the space as the product of the area of one of the walls and the distance to its parallel wall. Volume has the dimensions of area times length or length cubed and the unit used to measure volume is the cubic meter or something comparable.

Often we are more concerned with how fast something is moving rather than with the actual distance traveled. A driver on an interstate highway may constantly monitor the speedometer to avoid driving faster than 55 mph, but give little thought to the total distance traveled. The average rate at which distance is traveled is defined as the **average speed**. That is,

$$\textit{average speed} = \frac{\textit{distance traveled}}{\textit{elapsed time}} \tag{3.1}$$

As an example, suppose it takes you one-half hour to travel from one town to another town 20 kilometers away. Your average speed for the trip is calculated as:

$$\frac{20 \textit{ kilometers}}{0.5 \textit{ hour}} = 40 \textit{ km/hr}$$

In science, although the words are often used interchangeably, we draw an important distinction between speed and velocity. Average velocity is defined as the time rate of change of displacement. That is,

$$\textit{Average velocity} = \frac{\textit{change of displacement}}{\textit{elapsed time}} \tag{3.2}$$

Velocity, then, includes a magnitude and direction. As an example, suppose you travel 20 kilometers due west in half an hour. Your average velocity would be

$$\frac{20\ km\ due\ west}{0.5\ hour} = 40\ km/hr\ due\ west$$

Suppose you then return to your starting point in half an hour by traveling due east. Your average speed for the return trip is 40 km/hr and your average velocity is 40 km/ hr due east. Your average speed for the entire trip is 40 km/hr, but your average velocity for the trip is 0 km/hr because the net displacement is zero, since you returned to your starting point.

Average speed and average velocity do not specify how speed and velocity varied during your trip. For instance, consider if part of the trip was on a freeway where you traveled at 60 km/hr and part of the trip was through a town where your speed varied from 25 km/hr on some roads to 0 km/hr while you waited for a traffic light. To take into account these variations of speed, another concept, that of **instantaneous speed**, has been developed. Instantaneous speed is the speed measured at any instant of time. This is the speed which shows on your speedometer at any given moment. **Instantaneous velocity** (v) is the velocity at any instant of time, whose magnitude is the instantaneous speed at the same instant of time. Instantaneous velocity is more often of interest than is average velocity in discussions of acoustical systems.

An object experiences an **acceleration** (a) when its velocity changes, i.e., it speeds up or slows down. More precisely,

$$acceleration = \frac{change\ in\ velocity}{elapsed\ time} \qquad (3.3)$$

Velocity includes both speed and direction, so an acceleration can be obtained by either a change of speed or a change of direction, or by a change of both speed and direction.

3.3 Laws of Motion

Almost everyone has been pushed or shoved while standing at rest, so we have an intuitive grasp of the physical concept of **force** (F). If you push on a boulder you exert a force on the boulder and may cause it to move from its state of rest. Over 300 years ago Isaac Newton wrestled with the concepts of force and acceleration to formulate his three laws of motion. These important laws give us a physical definition of force.

Newton's **first law** states that a body in a state of uniform motion (or at rest) will remain in that state of uniform motion (or rest) unless acted upon by an outside force. The first law is a qualitative definition of force. It does not tell us what a force is but it does tell us when a force is not acting. If we observe an object at rest, we know there is no net force acting on the object. If you take this book out to the edge of our solar system and heave it as hard as you can off into space, it will travel forever in a straight line at a constant speed unless, it is influenced by another object.

While Newton's first law tells us when a force is or is not acting, Newton's **second law** tells us what happens when a force acts. When an unbalanced force acts on an object with mass, the object is accelerated (moved). The acceleration is directly proportional to the magnitude of the force and in the same direction as the force. When a force acts there is a change in the motion of an object. The force which caused the change is proportional to both the mass being accelerated and the resultant acceleration. We can represent this information symbolically as:

$$force = mass\ ?\ acceleration \qquad (3.4)$$

Newton's second law, then, is a quantitative definition of force. The law tells us that force is the product of mass and acceleration. The unit of force is a newton (N), which is defined as the force which causes a mass of 1.0 kg to accelerate at a rate of 1.0 m/s^2.

As an example of the second law, suppose that you wish to accelerate two different objects—a bicycle and an automobile—at the same rate. You must exert a much larger force on the automobile because of its much larger mass. When the small mass of the bicycle is multiplied by the acceleration we see that a small force is required. On the other hand, multiplying the large mass of the automobile by the acceleration results in a much larger force.

Newton's **third law** is probably the best known and yet least understood of the three laws. For every action (force) there is an equal and opposite reaction (force). The law tells us that forces do not occur singly in nature but in pairs. When two objects interact and object A exerts a force on object B, then object B exerts an equal and opposite force on object A. However, only one force in the pair acts on each object, and it is this force we must consider when applying the second law of motion to determine whether an object will be accelerated. For instance, when a violin string is bowed, the bow exerts a force on the string and the string exerts an equal and opposite force on the bow. Determining the acceleration of the string is done by considering only the force exerted on the string by the bow and not the force exerted on the bow by the string.

3.4 Additional Derived Quantities

It is easy to confuse the concepts of mass and weight, and people often use these words as synonyms. In science, however, weight and mass are defined differently, even though they are related concepts. Mass, as defined earlier, is a measure of the inertia or sluggishness a body exhibits when an attempt is made to change its motion in any way. **Weight** (w) is the force of gravitational attraction acting on a mass. Since your mass depends on the quantity of matter of which you are composed, it does not change if you visit the moon or Mars. Your weight on Mars, however, would be only 40% of your weight on Earth, because of the smaller gravitational attraction on Mars. Weight is a force, so it is measured in newtons (or pounds), just like any other force.

Which weighs more: a pound of lead or a pound of feathers? Don't be misled by confusing weight and density. A pound of lead and a pound of feathers have the same weight and the same mass, but the pound of feathers will certainly occupy much greater volume. Lead is a material in which massive atoms are very compactly grouped, while the less massive molecules which make up feathers are not very compactly grouped. The relative molecular masses and "degree of compactness" of different materials is expressed by the concept of density. The **density** (D) of a material is the mass (e.g., grams) occupied by a standard volume (e.g., cm^3) of the material:

$$density = \frac{mass}{volume} \tag{3.5}$$

The density of water is 1.0 gm/cm^3, while lead has a density of 11.3 gm/cm^3; air has the very low density of 0.0012 gm/cm^3.

A balloon filled with air will fly around a room when it is released. A metal can which has the air pumped out of it will be crushed by invisible forces. The concept of pressure is useful to understand the interaction between gases and container walls. **Pressure** (p) is defined as the force per unit area acting on the surface of an object. Thus,

$$pressure = \frac{force}{area} \tag{3.6}$$

The unit used to measure pressure is the pascal (Pa), defined as one N/m^2. As an example of the difference between pressure and force, consider a brick resting on its broadside on a table top. The pressure exerted on the table by the brick is the weight of the brick divided by the area touching the table. If the brick is turned on edge, the force (weight of the brick) remains the same, but because a smaller area is in contact with the table, the pressure is increased. If we upend the brick so its smallest face rests on the table, the pressure is even greater; the weight, of course, remains unchanged.

Why is a metal can crushed when the air is pumped out of it? Consider that we are living at the bottom of an ocean of air. In much the same way as the water behind a dam exerts a pressure on the dam, the atmosphere above us exerts its pressure (called atmospheric pressure) on all of us. Although the pressure is considerable (10^5 Pa at sea level), it usually goes unnoticed since we are pressurized from the inside. When the internal pressure is removed (as for the evacuated can) or even imbalanced slightly, the effects are noticed immediately. A scuba diver feels a crushing pressure deep under water. An uncomfortable feeling can occur in one's ears when one drives down a mountain or changes altitude suddenly in an airplane. The slight change of atmospheric pressure produces an unbalanced force on the eardrum which can only be relieved by equalizing the pressure.

3.5 Derived Electrical Quantities

Electric charge is an intrinsic property of electrons. As electrons move, their charge moves with them. Picture a stream of electrons and their associated electric charges moving from one location to another. A flow of electric charge is called an **electric current** (i) which is defined as the amount of charge passing a given point per unit time, that is:

$$current = \frac{charge}{time} \tag{3.7}$$

The unit of current is the **ampere** (A) defined as one coulomb per second. To maintain a continuous current requires a continuous supply of electrons on one end of a wire, a place for the electrons to go at the wire's other end, and an "electrical pressure." These requirements can be met by a battery. The flow of electrons from one end of the battery through the wire to the other terminal of the battery is like the flow of water through a pipe. To obtain a continuous flow of water through a pipe, a pump and a water supply must be provided at one end with an opening for the water to escape at the other end. The resulting flow of water is measured in cubic meters per second just as electric current is measured in coulombs per second. The total volume of water available is the number of cubic meters in the reservoir, just as the total electrical charge available is measured as the number of coulombs.

A pressure difference (supplied by a pump) must exist between two points in a pipe if water is to flow from one end of the pipe to the other. Similarly, an **electrical potential difference** (V) supplied by a battery must exist between two points in a wire if electrical current is to flow between them. The unit of potential difference is the volt (V) which is somewhat analogous to the pascal, the unit of pressure. We will see later in the chapter that analogies can be very useful for developing relationships among electrical quantities when we know the relationships among mechanical quantities and vice versa.

Just as two objects with mass exert gravitational forces on each other, two objects with electrical charges exert electrical forces on each other. When the two objects have like charges they repel each other; when they carry opposite charges they attract each other. The deflection of an electron stream in a TV set or a laboratory oscilloscope provides a practical example of electrical forces in action.

In addition to electric forces between charges at rest there are magnetic forces between moving charges. We will not define magnetism but will state some qualitative features about magnetic forces. When an electric current exists in a straight wire or in a loop of wire, a magnetic field is produced. A magnetic field is capable of exerting a magnetic force on any moving charge in its vicinity. (A permanent magnet may be thought of as due to many tiny atomic current loops that have all been oriented in the same direction to produce an effective "internal" current loop in the magnet.)

When a wire coil carrying a current is placed in a magnetic field it may be caused to move because of the magnetic force on its moving electrons; the greater the current the larger the force. Conversely, when a wire coil in a magnetic field is moved, current may be produced because of the magnetic force on the coil's electrons; the larger the motion, the greater the current. Electric motors, dynamic loudspeakers, and dynamic microphones are based on this principle.

When a wire loop is placed in a changing magnetic field a current is induced in the coil. This is the principle on which an electric generator is based.

3.6 Work

A detailed application of the concepts discussed earlier in this chapter would be too laborious to be useful at this point in your studies. The concepts of work, energy, and power may provide alternative descriptions for most things we find of interest in the physical aspects of music, speech, and audio. These new quantities can be defined in terms of the previously defined quantities, but in many ways they are handier to use.

In our everyday life we often evaluate the difficulty of a task in terms of the work involved. If a friend lifts a heavy object for you, he may complain that he is working too hard. If you push with all your strength against an immovable wall, you might claim that you are working equally hard. An observing scientist, although sympathetic to the effort you are exerting, would suggest that your friend is working, and you are not. The scientific definition of work takes into account the force exerted and the distance an object moves when that force is applied. The force must cause displacement of the object or the work done is zero. The **work** done by a force is the product of the force times the displacement in the direction of the force. Symbolically,

work = force ? displacement (in direction of force) (3.8)

Because you exerted a force on a wall which did not move, no work was performed. When your friend lifted the heavy object, however, the force exerted caused the object to be displaced upward and work was done. If we double the load that your friend lifts, we double the work done. Likewise, if the load is lifted twice as high, the work is doubled. The unit of measurement in which work is expressed is the product of distance and force units. In the metric system, the unit of work is the **joule** (J), defined as a newton-meter. One joule (rhymes with pool) of work results when a force of one newton displaces an object a distance of one meter in the direction of the force.

3.7 Mechanical Energy

"Energy" is perhaps the most fundamental unifying concept in all scientific disciplines. Yet, despite the prevalence of the term, it is an abstract concept which cannot be simply defined. You use "human energy" to turn the ignition key when you start a car. The ignition key engages the battery which converts chemical energy to electrical energy. The electrical energy from the battery is used in the starter to produce the mechanical energy which turns the flywheel which in turn starts the engine. The engine then converts chemical energy from fuel into heat energy. The heat energy turns into mechanical energy which propels the car. If it is dark you turn on the car's headlights, which convert electrical energy from the battery into light energy. If an animal runs in front of your car you honk the horn (converting electrical energy into sound energy) and step on the brakes (converting mechanical energy into heat energy). The basic points to remember in this hypothetical exercise are that (1) energy in some form is involved in all our activities, (2) energy can appear in many different forms, and (3) energy can be changed from one form to another. We will begin our discussion by considering mechanical energy, one of the most recognizable forms of energy.

Consider a simple frictionless pendulum (or swing) as shown in Figure 3.1. If we do work by applying a force to the pendulum we can move it from its rest position B to position A. If the pendulum is now released from position A it will gain speed until it reaches B, then lose speed until it reaches C, and then return through B to A. If the pendulum is truly frictionless the motion will repeat itself indefinitely, reaching the same height each time at A and C and having the same speed at B. There is an implication here that something is conserved, but that "something" can be neither height nor speed because both change throughout the motion.

We define the conserved quantity as **total mechanical energy,** energy because of position and energy because

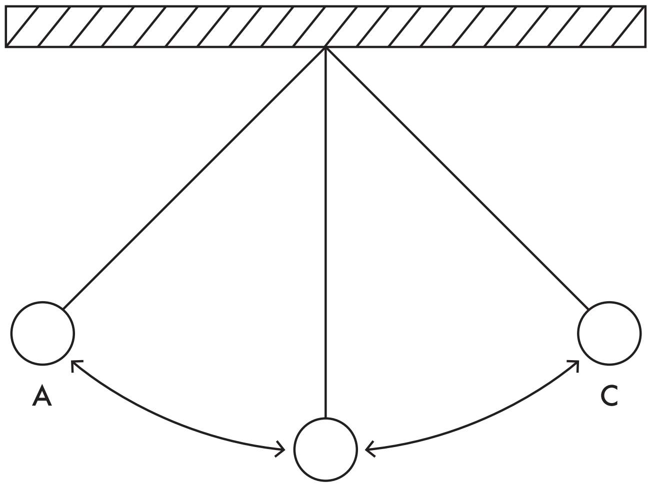

Figure 3.1 *Vibrating Pendulum*

of motion. The energy from position is termed **gravitational potential energy** expressed as

$$GPE = mgh = wh \tag{3.9}$$

where m is the mass of the pendulum, g is the gravitational constant, and h is the pendulum's height above its rest position. The GPE of the pendulum is just equal to the work done by a force (equal to the pendulum's weight, w, which is the product mg) in lifting the pendulum to height h above its rest position. The energy from motion is termed **kinetic energy**, expressed as

$$KE = \frac{mv^2}{2} \tag{3.10}$$

where m is the pendulum mass and v is its speed (magnitude of velocity) at any instant. The total mechanical energy of the pendulum is equal to the sum of KE and GPE and is conserved. In other words it is constant. As KE increases GPE decreases (and vice versa) so that their sum is always the same. As the pendulum goes from position A through position B to position C and back again, there is a continual transfer of energy from all potential (at A and C because the pendulum is not moving there) to all kinetic (at B where the speed is greatest). At the in-between points the energy is a combination of potential and kinetic, but at any point in the motion the sum of the potential and kinetic energy is exactly the same as at any other point.

Potential energy may also be explained with a stretched rubber band, a compressed spring, or a stretched drumhead. Potential energy associated with stretching or compressing objects is more relevant to our studies than is gravitational potential energy. For example, the **potential energy** of a spring is defined as

$$PE = \frac{sd^2}{2} \tag{3.11}$$

where s is a "stiffness" associated with the spring and d is the displacement magnitude of the spring from some rest position. In the preceding example of the pendulum the force required to lift the pendulum is constant (and equal to the weight of the pendulum). The force required to compress (or stretch) the spring, however, increases as the spring is compressed. The work done in compressing the spring is proportional to the displacement and to the force which in turn is proportional to the displacement. Hence, the work done in compressing the spring is proportional to the square of the displacement. Because the potential energy of the spring is just equal to the work done in compressing it, the potential energy of the spring is proportional to the square of the displacement. A mass attached to the spring will be accelerated and decelerated as the spring pushes and pulls on it. The kinetic energy of the mass at any instant will be

$$KE = \frac{mv^2}{2} \tag{3.12}$$

just as in the case of the pendulum.

The greater the amount of work done to raise a pendulum or to compress a spring, the greater its potential energy becomes. We have seen in the pendulum and spring examples that the potential energy in each case is just equal to the work done in raising the pendulum and in compressing the spring, respectively. We can then surmise that energy—potential, kinetic, or any other form—must be expressed in the same units as work, namely the joule.

Whenever a given physical quantity remains constant in a changing situation we generalize the result as a law. In this case the law states that the total mechanical energy (the sum of kinetic and potential energies) of a system remains constant when no frictional forces are present. In any real system there will always be some friction present and so this law is only an approximation of reality. However, it is still useful for analyzing vibrating systems, so long as its limitations are not forgotten.

3.8 Other Forms of Energy

What becomes of mechanical energy when friction is present? Consider the kinetic energy produced by a Girl Scout rubbing two sticks together. Where does this energy go? It turns into a different form of energy, one which we observe as heat in the sticks. Whenever mechanical energy disappears because of friction, **heat energy** appears. If we accurately measure the total heat energy produced and the total mechanical energy that disappears in a given situation, we would discover that the two amounts are equal; the mechanical energy lost equals the heat energy gained.

Likewise, heat energy can be changed to mechanical energy, as happens in a steam locomotive. But heat energy alone does not account for all possible energy transformations that take place. Suppose that you hammer a nail into a piece of wood. Some of the mechanical energy of the swinging hammer will be converted into heat (the nail gets warm). But what becomes of the remainder of the hammer's kinetic energy? It takes energy to deform the wood when the nail is driven in, and a sound is produced every time the hammer collides with the nail. Deformation and sound are other forms of energy which may appear when mechanical energy disappears.

Further experiments involving the transformation of mechanical energy to electrical energy (by means of a generator) or electrical energy to mechanical energy (by a motor) convince us that whenever one form of energy "disappears," equivalent amounts of some other energy forms "appear" in its place. The study of numerous transformations of energy from one form into another form has led to one of the great generalizations of science, the **law of conservation of energy**: energy cannot be created or destroyed, but it may be transformed from one form into another. The total amount of energy, however, never changes. The word "conservation" as used here does not mean to save for the future, but rather signifies that total energy always remains constant. Energy never appears or disappears; it is merely transformed.

The concept of energy conservation is very important in speech production, musical systems, and control of the energy level in an auditorium. Speech production and production of musical sounds require the performer to supply energy to the vocal mechanism or to the musical instrument. Part of the energy supplied by the performer is converted into acoustical energy that is emitted by the instrument as useful vocal or musical energy; part of the energy is lost internally due to frictional heating. Designers of listening rooms often purposely convert sound to other forms of energy to reduce loudness, or to ensure that the sound does not reverberate too long within the room.

3.9 Energy in Fluids

When we consider energy in fluids, we usually talk about energy density rather than energy per se. Kinetic energy density is then proportional to mass per unit volume times velocity squared. Potential energy density in a compressible fluid can be defined in terms of a "fluid stiffness" times the square of the fluid displacement in a manner analogous to that of a spring. However, potential energy density in a compressible fluid is more commonly defined in terms of the square of the pressure in the fluid. The crucial relationships for our purposes—whether we are studying about energy or energy density—are the dependence of KE on velocity squared and the dependence of PE on pressure squared.

Somewhat different relationships hold for energy density in an incompressible fluid (a fluid with no elasticity). When an incompressible fluid moves slowly its pressure is high and when it moves rapidly its pressure is low. This relationship accounts for the lift of airplane wings and the loss of house roofs from the action of tornadoes. It is also the basis for a driving mechanism of human vocal folds and lips, and mechanical reeds.

Consider the law of conservation of energy as it applies to water flowing through a pipe. When the water arrives at a constriction in the pipe, as shown in Figure 3.2, it must move faster to avoid piling up. (This effect would be analogous to three lanes of bumper-to-bumper cars moving at 20 km/hr and merging into one lane of traffic. If traffic is to keep moving so that a delay is avoided, each car must speed up to 60 km/hr at the constriction. If this unsafe method of merging traffic could be carried out, it would avoid further congestion.)

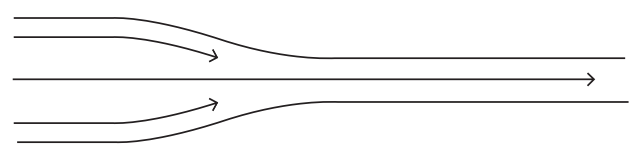

Figure 3.2 *Water flow in a constricted pipe*

When a flowing fluid reaches a smooth constriction, it increases its speed and the smooth flow continues with no congestion. But when the fluid gains speed, it also gains kinetic energy. Where does this energy come from? In the 18th century a Swiss scientist, Daniel Bernoulli, reasoned that the kinetic energy was acquired at the expense of decreased potential energy. One form of Bernoulli's law states that where kinetic energy density in a flowing fluid is large, potential energy density is small (and vice versa) so that the total energy density remains constant. Figure 3.3 illustrates this law for a pipe with a constriction. Notice that as the pipe is constricted kinetic energy density increases at the expense of potential energy density. The fluid moves more slowly as the pipe expands, and potential energy increases

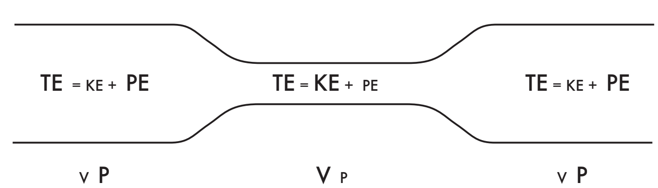

Figure 3.3 *Illustration of Bernoulli's law for fluid flow in a constricted pipe. Kinetic energy and velocity are large in the constriction. Potential energy and pressure are large in the unconstricted pipe. Total energy (TE) equals (PE) plus kinetic energy (KE) and is constant.*

at the expense of kinetic energy. For incompressible fluids the potential energy density is proportional to pressure (rather than pressure squared). Where the speed of moving fluid is large the fluid pressure is small and vice versa, as indicated by the labels at the bottom of Figure 3.3.

3.10 Power

In our discussion of energy transformations we did not discuss the important aspect of how rapidly energy can be transformed. One gallon of gasoline contains a certain amount of chemical energy, but the power we are able to produce will depend on how rapidly we use this energy. One gallon of gasoline in a power lawn mower yields ten horsepower for three hours while the same gallon of gasoline in an automobile may produce 60 horsepower for one-half hour. The same gallon of gasoline burned in a jet plane could produce 100,000 horsepower for one second! The difference is in the power of the engine, not in the energy content of the fuel. The concept of **power** (P) includes both the total energy expended and the time involved in expending it. More precisely:

$$power = \frac{expended\ energy}{elapsed\ time} \qquad (3.13)$$

Energy is expressed in the same units as work, namely joules. The unit of power is the **watt** (W) defined as a joule (energy) per second (time). This unit may be used for mechanical power, acoustical power, or electrical power. Just as energy is expressed in units of joules equal to newtons times meters, power may be expressed in units of watts equal to newtons times meters divided by seconds:

$$W = Nm/s \qquad (3.14)$$

Meters divided by seconds is a unit for velocity and so power may be expressed in units of force times velocity. Acoustical power can be expressed as pressure times "volume velocity." Electrical power can be expressed as voltage times current. The unit of power is the watt in all of these cases.

Consider the following example as an illustration of the difference between energy and power. Suppose you must exert a force of 400 newtons to climb ten meters up to the fourth floor of a certain building. One day you dash up the stairs in ten seconds. The next day you walk up the stairs in 100 seconds. After one particularly tiring day you crawl up the stairs in 1000 seconds. What was different in each of these cases? The work done by you was the same in each case because your weight and the distance traveled remained constant. The work done (or energy supplied) by you would be

$$E = F\ ?\ d = 400\ N\ ?\ 10\ m = 4000\ J$$

But the time it took you to do this amount of work was different in each case, so the power you had to supply was also different. The power exerted in the first case was

$$P = E/t = 4000\ J/10\ s = 400\ J/s = 400\ W$$

For the second case,

$$W = 4000\ J/100\ s = 40\ W$$

and for the third case,

$$W= 4000\ J/1000\ s = 4\ W$$

Thus, the longer it took you to travel up the stairs the less the power you required. The total work or energy in each case was the same because, even though less power was supplied, it was being applied for a longer time.

The concept of power has many important applications in the science of acoustics. When we discuss hearing acuity, we will be concerned with the sound power perceived by an ear. When noise problems are discussed, they are rated by their overall sound power. Musical instruments are judged to be loud or soft based on their power output, and stereo amplifiers are rated by their power output in watts.

The wattage rating of electric light bulbs is a power rating; a 200-watt bulb requires twice as much power, or twice as much energy per second, as a 100-watt bulb. The power we encounter when we deal with sounds is much smaller than typical electrical powers, which testifies to the remarkable sensitivity of the human ear. But even so, the same principles can be used to rate in watts the sound output of various sources. Consider, for example, a 100-watt light bulb. While perfectly adequate to provide light for an average-sized room, it is not a very powerful source of light. Yet those familiar with audio equipment will know a 100-watt amplifier is enough to rock an entire house! As a matter of fact, a 10-watt amplifier connected to typical loudspeakers is capable of producing more sound than many adults would tolerate.

3.11 Intensity

How are we to account for the fact that as we move farther from a sound source such as a loudspeaker its sound diminishes? Even though a loudspeaker produces constant sound power we perceive the sound as less loud when we move away from the loudspeaker, especially outdoors where there are no walls to reflect the sound. A similar effect occurs with a 100-watt light bulb. The power output of the light bulb remains the same, but somehow we perceive it as less bright as we move away.

Imagine 100 watts of sound power spreading out in all directions in an ever-enlarging sphere. The total sound power remains constant on the surface of the sphere, but it is spread over a larger area as the size of the sphere increases. The **intensity** (I) of a sound wave is the power per unit area; that is, the total sound power divided by the surface area through which it passes. The unit of intensity is watt per square meter (W/m^2). Figure 3.4 is a representation of sound power being spread over larger and larger areas as it travels away from the source. As we move away from the source, the sound becomes less intense (i.e., the same power spreads over a larger area) and is perceived as being softer. In Figure 3.4 we can see that if a certain amount of sound power falls on a 1 m^2 surface located at a distance of 10 m from the source, the same sound power would be spread over 2 ? 2 = 4 m^2 at a distance of 20 m and 3 ? 3 = 9 m^2 at a distance of 30 m. The intensity decreases as the inverse square of the distance from the source; when the distance from the source is doubled, the sound intensity falls to one-fourth its previous value. (This assumes that there are no reflecting walls and that the sound spreads out uniformly in all directions.)

We can find other useful relationships of proportionality among energy, power, intensity, and pressure. We have noted previously that energy and energy density are proportional to pressure squared, albeit with different coefficients of proportionality. Power is proportional to energy and intensity is proportional to power, so power and intensity are proportional to pressure squared, again with different coefficients of proportionality. We have just seen that intensity varies with the inverse square of the distance so that intensity reduces to one-fourth its previous value with each doubling of the distance. However, because intensity varies with the square of the pressure, the pressure must reduce to one-half its previous value with each doubling of the distance.

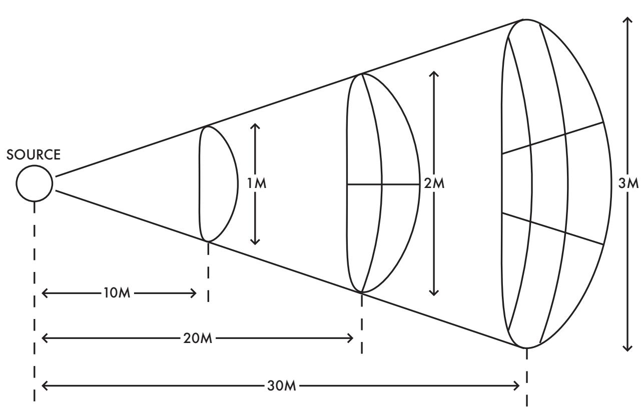

Figure 3.4 *Diagram illustrating the decrease of intensity with increasing distance from the source. The sound power is spread over a large area at greater distance from the source. The sound is assumed to spread out uniformly in all directions, so the diagram shows only a portion in a small solid angle.*

In addition to the decrease of sound intensity due to "spreading out," sound energy and intensity may be decreased because of the absorption of sound energy when a sound wave contacts an object and because of the transformation of sound energy into heat energy as a sound wave moves through air. These causes of energy loss are important for rooms and will be discussed in Part III. The absorption of sound energy at the walls of wind instruments can be significant, often consuming in excess of 90% of the energy supplied by the player. Energy is also lost from musical instruments by radiation from strings, membranes, or toneholes.

3.12 Summary

All physical quantities used in science can be traced back to a few fundamental units. Length, mass, time and electrical charge can be combined to describe quantities such as velocity, force, work, and energy. Speed and velocity differ in that velocity has direction associated with it. Newton's three laws of motion describe how masses react when forces are applied to them. The law of conservation of energy states that energy cannot be created or destroyed, but it may be transformed from one form into another. Intensity is defined as the power per unit area and provides an explanation for why sound becomes softer at farther distances from a source.

References and Further Reading

Backus, J. (1977). *The Acoustical Foundations of Music, 2nd ed.* (Norton). Chapter 1 is a good alternative presentation of the material in this chapter.

Serway, R. A. and J. W. Jewett Jr. (2006). *Physics for Scientists and Engineers, 6th ed.* (Brooks/Cole). Chapters 1–14 provide a more technical description of the material presented in this chapter.

Questions

3.1 Determine which of the following statements and questions are meaningless: Are there natural laws which man can never hope to discover? All matter and space are permeated with an undetectable substance. The entire universe is expanding so that everything in it has all its linear dimensions doubled every month. Is the sensation which I experience when I see green the same as that which you experience when you see green?

3.2 Describe experiments which would demonstrate that there are two and only two types of electrical charge.

3.3 According to Newton's first law, once an object is started in motion it will continue in a straight-line motion forever if no forces act on it. Explain why objects we start in motion (by sliding, or throwing, etc.) always come to a stop.

3.4 A 50 kg boy and a 100 kg man stand in identical carts on a level surface. Each is pushed with equal force. Explain what happens and why.

3.5 Explain how a moose can pull a cart across a horizontal surface when the force of the cart on the moose is equal in strength but opposite in direction to the force of the moose on the cart. Consider all horizontal force pairs, but ignore vertical force pairs.

3.6 Suppose you are in an orbiting satellite where everything is "weightless." If you have two identical cans, one empty and one filled with lead, what experiments could you perform to determine which has the greater mass?

3.7 Explain (in terms of electrons) why charging objects by rubbing them together (static electricity) always yields equal amounts of positive and negative charge.

3.8 Distinguish between 10 N and 10 N/m^2.

3.9 How would the air density at the earth's surface compare to the air density at the bottom of a deep mine? On top of a high mountain? Explain.

3.10 Why doesn't atmospheric pressure crush us?

3.11 A pumpkin held above the ground has potential energy. If the pumpkin is dropped, what becomes of the potential energy just before it hits the ground? After it hits the ground?

3.12 Why does a swinging pendulum eventually cease its motion? What becomes of the mechanical energy?

3.13 Where in its motion is the kinetic energy of a pendulum bob at a minimum? Where is its kinetic energy at its maximum? When the potential energy is half of its maximum value, what is the kinetic energy?

3.14 Why does your car tend to lurch toward the middle of the road when you pass an oncoming truck on the highway? As your relative speeds increase, will the effect increase or decrease?

3.15 When you drive a car along a road at a constant speed, the engine burns fuel continuously and does work on the car, and yet the car does not gain kinetic energy. Explain.

3.16 When you raise a book above a table you do work on the book. When the book is at rest one meter above the table top, in what form is the energy and from where did it come? When you drop the book, in what form is the energy the instant before the book hits the table? When the book hits the table and comes to rest, what happens to the energy?

3.17 Why do automobile brakes get hot when stopping the car?

3.18 What type of energy is the electrical energy being changed to in a light bulb? An electric motor? An oven?

3.19 Would it be correct to state that all changes in the physical world involve energy transformation of some kind? Explain.

3.20 Would it be possible to hurl a snowball against a wall at such speed that the snowball would completely melt upon impact? Explain.

3.21 A device has been proposed which has a grinding wheel hooked to an electric motor that also runs a generator, which in turn generates the electricity to turn the motor. Without knowing much about generators and motors, why would you conclude that such a device will not work?

3.22 Why does an electric motor require more current when it is started than when it is running continuously?

3.23 An oboe is found to be only 3% efficient in converting the player's input energy into useful acoustic energy that is radiated as sound. What happens to the rest of the energy? Where is it lost?

3.24 Two excellent musicians (an oboist and a trumpeter), with comparable lung capacities, have a contest to see who can blow their instrument for the longer time. Both performers are able to sustain a continuous tone for the same time period, so the result of the contest is a tie. However, people who listen to the sound output observe that the trumpet tone is consistently louder than the oboe tone. Does this observation offer any clues as to the relative efficiencies of the two instruments? Explain.

Exercises

3.1 Compute the average velocity in each of the following cases. An auto moves 60 m west in 100 s. A bicycle moves 1 cm in 1 s. A person walking moves -0.1 cm in 10^{-3} s.

3.2 Compute the average acceleration for each of the following cases. The speed of an auto changes 6000 cm/ s in 10 s. The speed of a bicycle changes -1.0 cm/s in 10^{-2} s.

3.3 Instantaneous velocity can be determined by taking a very small change in displacement (represented as Δd) and dividing it by the very small elapsed time (Δt) to give v = d/Δt. Assume that a vibrating string moves a distance of 0.001 cm in 0.000 1 s and calculate the instantaneous velocity. (The Δ notation is used to indicate a small change of a quantity such as time, distance, etc.)

3.4 The relationship among instantaneous velocity, displacement, and time can be written v = Δd/Δt. If Δd = 0.50 cm and Δt = 0.01 s, find the instantaneous velocity. If v = 100 cm/s and Δt = 0.05 s, find Δd. If v = 100 cm/s and Δd = 0.20 cm, find Δt.

3.5 The instantaneous acceleration of an object can be written: a = Δv/Δt. When Δv = 0.10 cm/s and Δt = 0.05 s, find a. When a = 6.0 cm/s^2 and Δt = 0.05 s, find Δv. When a = 5.0 cm/s^2 and Δv = 0.10 cm/s, find Δt.

3.6 Imagine that you are driving your car on a perfectly straight highway. Calculate your acceleration for each of the following situations. You increase your speed from 20 km/hr to 40 km/hr in 10 s. You decrease your speed from 40 km/hr to 20 km/hr in 5 s. You remain at a constant speed of 30 km/hr for 20 s.

3.7 Explain why spiked heels will puncture a hard floor while a normal heel will not. As an example, compare the pressure exerted by a 50 kg woman wearing 1 cm^2 heels to that of a 100 kg man wearing 50 cm^2 heels

3.8 The blowing pressure in a clarinet player's mouth is 2000 N/m^2. What force is exerted on a clarinet reed area of 1 cm^2 by the blowing pressure. (The reed area should be converted to m^2 before calculating the force.)

3.9 Air near sea level has a mass density of 1.3 kg/m^3 and a corresponding weight of 12.7 N/m^3. Although the density of the atmosphere decreases with altitude, the total amount of air is equivalent to a column of "sea level air" about 7900 meters high. What is the pressure of this air column (expressed in N/m^2 and Pa)? This pressure is referred to as atmospheric pressure, which as you know changes with altitude and with changing weather patterns.

3.10 How high a column of water would be required to produce a pressure equivalent to that of the atmosphere? (Water has a density of approximately 1000 kg/m^3.)

3.11 Calculate the potential energy of a spring with a spring constant of s = 500 N/m stretched 5 cm from its rest position.

3.12 If the spring in Exercise 3.11 is stretched in 0.5 s, what power is required?

3.13 How much work do you do in pushing your bicycle 100 m up a hill if a constant force of 10 N is required?

3.14 If the bicycle of Exercise 3.13 is pushed up the hill in 10 s, what power is required? How much power is required if 1000 s is taken?

3.15 If a clarinet reed is moved 1 mm under a force of 0.2 N, how much work is done? If the reed vibrates 500 times per second and the work is done during one-quarter of a vibration, what power is required?

3.16 Total energy of 500 J is received during a time of 10 s. What is the power? If the receiving surface has an area of 10 m^2, what is the intensity?

3.17 A microphone diaphragm has a diameter of 10 mm. What power (in watts) does it receive from a sound having an intensity of 10^{-5} W/m^2? How much energy does it receive in 25 s?

Activities

3.1 Demonstration of atmospheric pressure: Watch a can get crushed by the atmosphere as it is evacuated. Try to pull apart the two halves of an evacuated spherical shell.

3.2 Demonstration of energy conservation: Make a pendulum by suspending a large mass on a light string. Set it into motion. Observe the time required for the motion to cease. What becomes of the energy?

3.3 Repeat Demonstration 3.2 for a mass on a spring.

3.4 Demonstration of Bernoulli's law: Place a Ping-Pong ball inside a funnel through which air is streaming upward through the narrow end. The ball will not be blown up and out of the funnel. Now invert the funnel and the ball will be held in position rather than blown away by the air. The airflow around the ball is constricted and the pressure is consequently less than the atmospheric pressure below the ball. The ball is actually pushed into the region of lower pressure above it by the greater pressure below it, so that it remains supported in midair.

3.5 The Bernoulli force can be demonstrated by placing a small card (3 ? 5) on a table. Place a thread spool on end and centered on the card. Use a tack pushed through the center of the card to keep it centered on the spool. Blow through the spool and lift it away from the table. The card will also come.

3.6 Get identical spherical balloons. Blow one up to twice the radius of the other. Paste stickers on the balloons to just cover their surfaces. How many times as many stickers are required to cover the large balloon? Total balloon material can be thought of as total sound power. The larger a balloon becomes as it is blown up the less balloon material there is per unit surface area—power per unit area or intensity.

CHAPTER 4

Properties of Simple Vibrators

What do a dog's wagging tail, a swinging clock pendulum, and a sounding trumpet have in common? Vibration is the common element, and the study of vibration will engage our attention throughout the remainder of this book. All sounds ultimately result from and consist of mechanical vibrations. We will first consider only periodic motion, motion that repeats itself over and over again in equal time intervals. Later, as our studies lead to more and more complex vibrating systems, we will broaden our perspective to include nonrepeating vibratory motions. **Periodic motion** may consist either of a simple repetitive pattern or a complex repetitive pattern. In this chapter we will consider the simplest of all periodic motions, simple harmonic motion.

4.1 Simple Harmonic Motion

Consider the swinging pendulum in Figure 3.1 as an example of simple periodic motion. The pendulum hangs straight down and is "centered" when at rest. As it oscillates or swings, it moves from left to right and back again. If we label the center as the "rest position," we may speak of the "displacement from the rest position" as the distance left or right from the center to the position of the pendulum at any given time. This information would be more useful, however, if it could be graphically represented. Suppose we attach a small marker to the bottom of the pendulum and pull a strip of paper at a constant rate beneath the oscillating marker so that a curve is traced on the paper as indicated in Figure 4.1. Examine the curve produced by the swinging pendulum. The distance along the paper strip represents time. The back and forth distribution of the marker's ink represents the left or right displacement of the pendulum from its rest position. Although this curve was produced for one particular motion, experimentation with any similar repetitive motion would produce a similar characteristic curve.

Any object that is capable of vibrating must have a rest position where it remains while not vibrating. Such a position is called the **equilibrium position** because the sum of all forces acting on the object at this point is zero. Any disturbance of the object will cause it to move away from this position to some new position in which the forces on the object no longer add up to zero. The object then experiences a "restoring force" which pulls it back toward its equilibrium position. This restoring force may be due to gravity, to internal stiffness (elasticity) of a material, or to an externally applied tension. As the restoring force pulls the object back toward its rest position, the object eventually returns to the rest position, but in doing so it acquires speed which causes it to overshoot and travel some distance to the other side of its equilibrium position.

The object then experiences a new force that again pulls it back toward the rest position, but it again overshoots the rest position. This back and forth motion continues to repeat itself. Motion of this general type is called

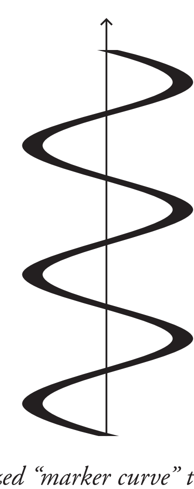

Figure 4.1 *Idealized "marker curve" traced out on a moving strip of paper by a swinging pendulum. The head of the arrow shows direction of paper motion; the arrow body shows the equilibrium position of the pendulum.*

an oscillation or vibration. It is characterized by the fact that the farther the object moves from the equilibrium position, the greater the restoring force which acts to pull the object back. For a pendulum, F = mgx/L, where F is the restoring force, m is the mass, L is the length, x is the displacement from rest position and g = 980 cm/s^2 is the acceleration due to gravity. If the restoring force is exactly proportional to the distance from the rest position as in this example, we have the special type of oscillation known as **simple harmonic motion** (abbreviated SHM).

Simple harmonic motion is characterized by five special attributes:

(1) The motion continually repeats itself in a definite interval of time, called the period; the **period** (T) is the amount of time required for one complete oscillation (positive and negative swing).

(2) The motion is symmetric about the equilibrium position. The maximum displacement to either side of the rest position is identical.

(3) The displacement curve traced out in time by an object undergoing SHM is a special curve called a **sinusoid**.

(4) The velocity of the object varies continuously throughout the oscillation. The velocity must be zero at the positions of extreme displacement where the object momentarily stops. It becomes maximum when the object passes through the rest position. The velocity varies in a symmetric, repetitive manner and can also be described by a sinusoidal curve.

(5) The acceleration of the object also varies sinusoidally.

As another illustration of SHM, consider a mass that is permanently attached to one end of a spring as in Figure 4.2. The spring is placed in a horizontal position on a frictionless tabletop (e.g., an air table), with the other end fastened to a rigid support. Figure 4.2 is then a top view of this arrangement and shows that the mass is free to slide on the table. The force and displacement are related by F = –sx where F is the restoring force, s is spring stiffness, and x is the displacement from equilibrium. We can start the mass oscillating by first displacing it (which stretches the spring) directly away from the table support and then releasing the mass. The mass will move toward the support, eventually compressing the spring. The mass will then move away from the support because of the force of the compressed spring. The one-dimensional motion toward and away from the support will repeat itself. (We are confining our attention to vibration in one dimension. You can imagine other vibrations with the mass swinging back and forth like a pendulum or twisting the spring to produce torsional motion. In many practical situations we can describe a vibrator adequately in terms of one-dimensional motion. We will use this approach unless stated otherwise.)

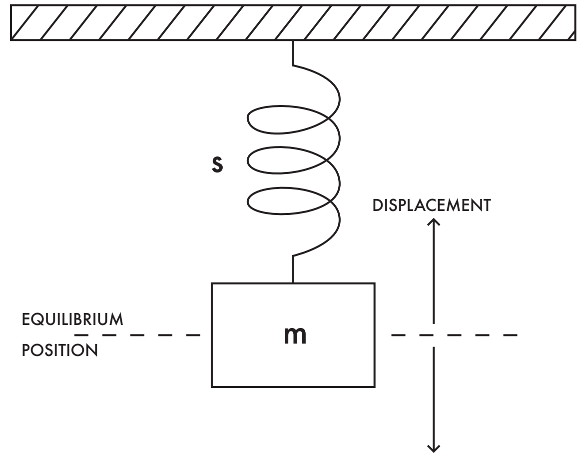

Figure 4.2 *Mass and spring vibrator with mass m and stiffness s.*

Suppose that we mount a movie camera directly above the oscillating mass of Figure 4.2 and film the motion at a rate of ten frames per second. The movie film provides us with a detailed record of the location of the mass relative to its rest position at time intervals of 0.1 s. To obtain the displacement of the mass from its rest position at each time interval we need only measure the appropriate distance as recorded in each frame of the film. Some of these measured data are recorded in the displacement column of Table 4.1. The initial frame (labeled 0) was arbitrarily chosen from one of the frames where the mass was passing through its rest position. The displacement data from Table 4.1 can be used to construct the upper graph of Figure 4.3, which shows how the displacement varies with time. The displacement has both positive and negative values because it is

Table 4.1 *Values of displacement and velocity for a mass and spring vibrator.*

Frame	Time (s)	d (cm)	v (cm/s)
0	0.0	0.00	6.30
1	0.1	0.59	5.10
2	0.2	0.95	1.95
3	0.3	0.95	-1.95
4	0.4	0.59	-5.10
5	0.5	0.00	-6.30
6	0.6	-0.59	-5.10
7	0.7	-0.95	-1.95
8	0.8	-0.95	1.95
9	0.9	-0.59	5.10
10	1.0	0.00	6.30
11	1.1	0.59	5.10
12	1.2	0.95	1.95
13	1.3	0.95	-1.95
14	1.4	0.59	-5.10
15	1.5	0.00	-6.30
16	1.6	-0.59	-5.10
17	1.7	-0.95	-1.95
18	1.8	-0.95	1.95
19	1.9	-0.59	5.10
20	2.0	0.00	6.30

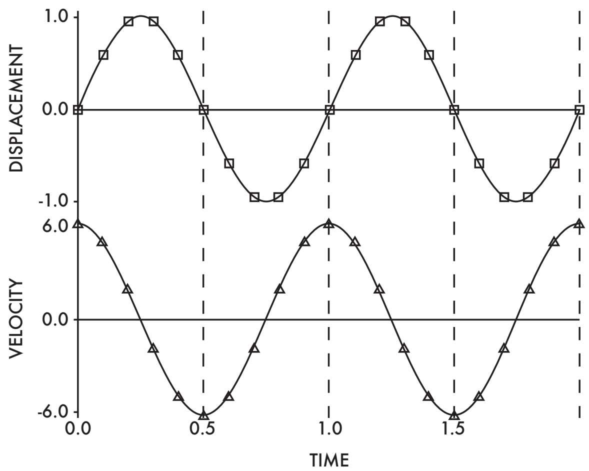

Figure 4.3 *Displacement and velocity for the mass and spring vibrator.*

measured either "above" or "below" the rest position. After the data are plotted, the individual points might be connected by a smooth curve (the solid line in the graph). The graph clearly shows how the sinusoidal curve is produced by plotting the displacement of the mass versus time.

The column of velocities in Table 4.1 was computed from the frames of the movie film in order to obtain the instantaneous velocity of the mass. A positive velocity represents motion of the mass upward even if the mass is below the equilibrium position. A negative velocity represents downward motion of the mass. A plot of the velocity versus time is shown in the lower graph of Figure 4.3. You will note that the curve of velocity versus time is also a sinusoid, but it is shifted with respect to the displacement curve. When the displacement is zero, the velocity is at a maximum, i.e., as the object passes through the equilibrium position (displacement zero) it is traveling at its maximum speed in either the positive or negative direction. On the other hand, when the displacement is at a maximum (either positive or negative), the object stops momentarily and, thus, its velocity is momentarilyl zero

In summary, when a mass is undergoing SHM, the curves representing the manner in which the displacement and the velocity change with time are sinusoids.

4.2 Comparing Simple Harmonic Motions

The foregoing discussion has considered features that are common to all simple harmonic oscillators. However, not all simple harmonic motions are identical; they may differ in size, repetition rate, and relative starting times.

Figure 4.4 show the displacement of the mass in a simple vibrator for several cycles. The maximum displacement in either direction from the rest position is known as the

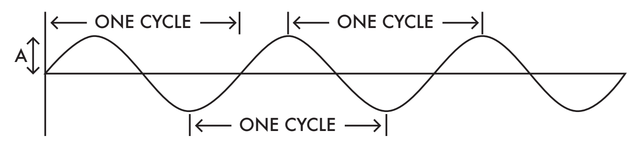

Figure 4.4 *The amplitude and several ways of defining a cycle for SHM.*

displacement amplitude (A). The total displacement from the maximum on one side of the rest position to the maximum on the other side of the rest position (sometimes called the peak-to-peak displacement) is thus twice the amplitude. In general, for any sinusoid, the amplitude is the distance from the rest position to a peak of the sinusoid. The velocity amplitude refers to the maximum velocity, which occurs when the object passes through the equilibrium position.

One cycle of a vibration is defined as one complete excursion of the mass from the rest position over to one extremity, back through the rest position to the other extremity, and then back again to the rest position. One cycle of a sinusoid can be used to represent one complete oscillation. It is not mandatory that a cycle be measured from the equilibrium position. Any point of the sinusoid is a valid starting point, but one cycle is that portion of the sinusoid between the starting point and the next identical point on the sinusoid.

It is often more convenient when considering a vibrating object to speak of how frequently it repeats its motion than to speak of the time required for completion of one cycle defined previously as the period (T). The term **frequency** (f) is used to express the number of cycles completed in each second. The unit of frequency is the **hertz** (abbreviated Hz and representing one cycle per second). Because frequency is the number of cycles occurring each second and the period is the number of seconds required for one cycle, it can be seen that they are reciprocally or inversely related as $f = 1/T$; when one increases, the other must decrease. For example, if a vibrating mass on a spring has a frequency of 10 Hz, its period would be 1/10 s or 0.1 s; if the frequency of vibration is decreased to 2 Hz the period is increased to 0.5 s.

If we displace two identical mass and spring systems the same distance and release them together (Figure 4.5), they will oscillate exactly "in step." Their motions are identical in every way; they have the same amplitude and the same frequency. If we displace the masses by the same amount, but in opposite directions before releasing them, they will still oscillate with the same frequency and the same amplitude (Figure 4.6), but they will oscillate exactly "out of step"—when one is moving up, the other is moving down. The difference between the motions illustrated in Figures 4.5 and 4.6 is expressed in terms of phase.

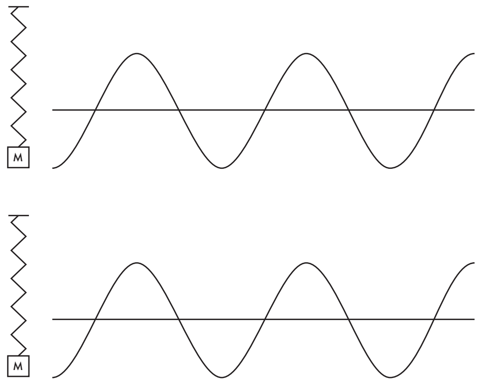

Figure 4.5 *Two identical mass and spring systems vibrating in phase.*

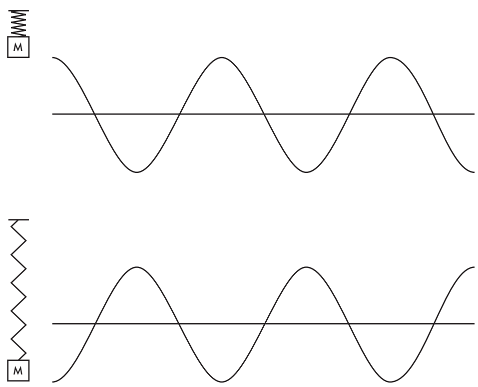

Figure 4.6 *Two identical mass and spring systems vibrating out of phase.*

Phase is the fraction of a period (or cycle) between some reference point and any other point on a sinusoid. Phase is commonly expressed by reference to a point moving around a circle as seen in Figure 4.7. One complete trip of the point around the circle is one period and corresponds to 360°. (Phase can also be expressed in radians where 2π radians is equal to 360°.) Hence, a phase of one period is equivalent to 360°, a phase of one-half period is equivalent to 180°, a phase of one-quarter period is equivalent to 90°, and so on.

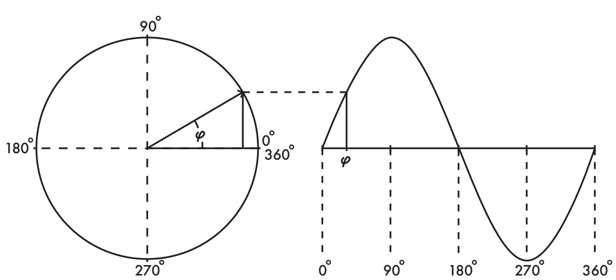

Figure 4.7 *The vertical projection of a point moving around a unit circle gives the value of the sine functions. Its angular position gives the phase of the sine function in degrees. Note that 360° corresponds to one complete period after which everything repeats.*

Phase difference is the phase relationship between two sinusoids. One way to express phase difference between sinusoids is in degrees or fractions of a period. For example, when two masses are moving in step (Figure 4.5) the phase difference is zero and they are said to be in phase. When two masses are moving opposite to each other (Figure 4.6) the phase difference is 180° or one-half period and they are said to be 180° out of phase. Note that the lower curve of Figure 4.6 could be made identical to the upper curve if it were shifted to the left (or right) by one-half period. The phase difference between the displacement and velocity curves shown in Figure 4.3 is 90° or one-quarter of a period.

It is possible to write an expression for the instantaneous displacement of a simple vibrator in terms of a sinusoid as

$$d(t) = A \sin (360\, t/T + ?) \qquad (4.1)$$

where A is displacement amplitude, t is elapsed time, T is the period, ? is the phase when time is zero, and sin is the sine function. The quantity (360t/T + ?) forms the argument (expressed in degrees) for the sine function, values of which are tabulated in Appendix 6. (Hand calculators can also be used to find the value of the sine function when the argument is expressed in degrees or radians.) We note that every time t increases by T this argument increases by 360°. The above expression can be written in a more convenient form by realizing that f = l/T so that

$$d(t) = A \sin (360\, f\, t + ?) \qquad (4.2)$$

The argument (360 f t + ?) can be interpreted as number of degrees where 360 f t and ? are each expressed in degrees. As an example, we use Equation (4.2) to calculate the displacement of a mass and spring vibrator having an amplitude A = 5 cm and a frequency f = 2 Hz. We assume that ? = 0 when t = 0 and want to know the displacement d when t = 0.05 s. Thus, 360ft = (360)(2)(0.05) = 36°, sin (36°) = 0.59, and d = 5 sin(36°) = 5(0.59) = 2.95 cm.

4.3 Effect of Mass and Stiffness on SHM

Small objects generally vibrate rapidly while large objects tend to vibrate more slowly. The wings of a mosquito oscillate rapidly enough to produce a high-pitched hum, whereas an earthquake may jolt the earth and cause undulations having a period of one hour or longer. What is the explanation for the generally different vibration rates of small and large bodies? The explanation does not depend upon size but rather upon mass—the more massive the vibrator the slower its natural vibration. Consider again a mass attached to a spring and vibrating with SHM. If we double the mass, the same spring force must change the motion of twice as much mass. Because the increased mass has more resistance to changes of motion (inertia), these changes take place more slowly, resulting in a longer time to complete one oscillation. Thus, increasing the mass of a vibrator increases the period of vibration and decreases the frequency. Conversely, reducing the mass of a vibrator reduces the period and increases the frequency.

What would happen if, instead of changing the mass, we exchanged the spring for a stiffer one? The stiffer spring, when displaced from equilibrium, will exert a greater restoring force on the mass, thus producing more rapid changes of motion. The mass completes its motion more rapidly, the period is reduced, and the frequency is increased. Increasing the stiffness of a simple harmonic oscillator results in an increased frequency while decreasing the stiffness gives a corresponding decrease in frequency. The **stiffness** (s) of a spring is measured by the **spring constant**. The greater the force required to stretch a spring by a certain amount, the greater the spring constant will be. A stiff spring has a large spring constant (large stiffness) while a "soft" spring has a small spring constant.

A simple mass-spring system set into motion and allowed to vibrate freely exhibits SHM; its (sinusoidal) pattern of vibration is called its natural mode. The frequency with which the simple mass-spring system vibrates freely is called its natural frequency. Natural frequency is directly proportional to the square root of the spring stiffness:

$$f \sim \sqrt{s} \tag{4.3}$$

Natural frequency is inversely proportional to the square root of the mass:

$$f \sim \sqrt{\frac{1}{m}} \tag{4.4}$$

Combining the foregoing relationships with constants of proportionality expresses the dependence of natural frequency on mass and spring stiffness as

$$f = \frac{1}{2\pi}\sqrt{\frac{s}{m}} \cong 0.16\sqrt{\frac{s}{m}} \tag{4.5}$$

4.4 Damped Free Oscillation

In our discussion so far we have been implicitly assuming that once we start an object vibrating, it continues oscillating indefinitely. Such, of course, is not the case. If, for example, the freely vibrating mass and spring of Figure 4.2 are placed on a table where there is even a small amount of friction, the vibration amplitude will decrease slowly as illustrated in the upper part of Figure 4.8. When the system is subjected to a very large resistance, such as would

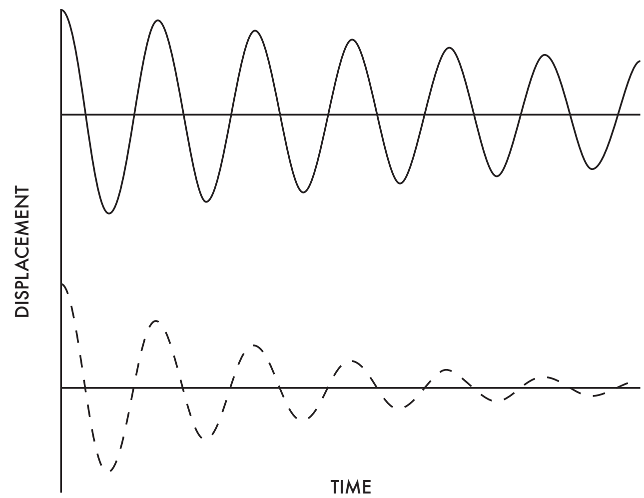

Figure 4.8. *Displacement versus time for a free vibrator with small resistance (solid line) and a free vibrator with large resistance (dashed line).*

result from a rough surface, the amplitude decreases more rapidly, as can be seen in the lower part of Figure 4.8. **Damped oscillation** results when the amplitude of an oscillator decreases with time. Free oscillators will eventually come to rest if no means is provided to sustain their oscillation because oscillatory energy is lost to the various forms of friction which are always present. In a mechanical system, such as the mass on a spring, friction converts mechanical energy into heat. In musical instruments acoustical energy is converted into heat by several forms of friction. In all cases, the vibration energy decreases with time; systems with more friction lose their energy more rapidly.

A convenient way to express the rapidity with which amplitude decays is the time required for the amplitude to decrease to one-half its initial value. This is referred to as the **damping time**. An interesting characteristic of the damping time is that it is independent of the initial amplitude. The amplitude decreases by equal fractions in equal time intervals. If the initial amplitude, A, decreases to A/2 during the first second, the amplitude will decrease to 1/2 of A/2 or A/4 during the second second, and to A/8 during the third second. This is illustrated in Figure 4.9. The ini-

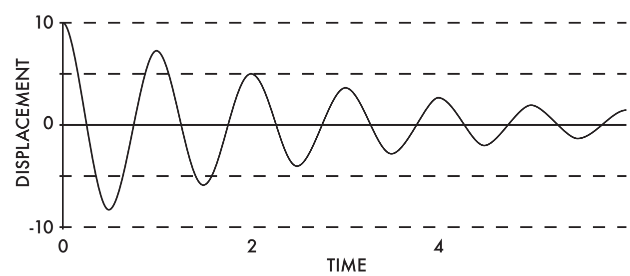

Figure 4.9 *Illustration of equal fractional decrease in amplitude in equal time intervals for a free vibrator.*

tial amplitude for this case is 10 cm, and the damping time is 2.0 seconds. Note that every two seconds the amplitude has become one-half its previous value. These concepts are particularly important to the understanding of percussion instruments discussed in later chapters.

For the strings and membranes used in most percussion instruments, the damping time is sufficiently long so that once set into motion the vibration continues long enough for the instrument's intended purpose. Examples are a piano string, a guitar string, and a drumhead.

4.5 Driven Oscillation

In nonpercussion instruments the oscillations would die out too rapidly for intended musical purposes if energy were not being supplied continuously by the performer. We distinguish then between a **free vibrator**, which is any vibrator set into oscillation and then allowed to decay, and a **driven vibrator** to which energy is continuously supplied. In free vibration the object vibrates at its own natural frequency, which remains constant even though the amplitude decreases continuously. In forced vibration, when a vibrator is driven by a continuous sinusoidal force the vibrator will vibrate at the same frequency as that of the driving force. For instance, if a sounding tuning fork is held over the opening of a bottle, a feeble sound of the same pitch as the fork will be heard from the air in the bottle. The vibration of the air is driven by the periodic sound waves emitted by the tuning fork. Blowing across the mouth of the bottle will result in a much louder sound of different frequency, as this corresponds to the free vibration of the air in the bottle. (The air in the bottle supplies stiffness and the air in the neck of the bottle supplies mass.)

Suppose a sinusoidal force having a frequency of 0.5 Hz is applied to a mass on a spring having a natural frequency of 1.0 Hz. After an initial period of instability the vibrator will settle down to vibrating at the driving frequency of 0.5 Hz, but with a relatively small amplitude. The displacement and the driving force will be in phase—when the force pushes upward, the mass moves up. If we now raise the frequency of the driving force to 2.0 Hz, the mass will vibrate with a frequency of 2.0 Hz and with a small amplitude, but with one important difference: the displacement and the force are now one-half cycle out of phase! The force pushes up while the vibrator is moving down and vice versa. It is a general characteristic of driven vibrating systems with small damping that when the driving frequency is below the natural frequency, the displacement is in phase with the driving force, while above the natural frequency they are out of phase.

4.6 Resonance

Imagine a mass and spring vibrator whose natural frequency is 10 Hz. Suppose that the frequency of a driving force is started well below the natural frequency and increased to some frequency well above the natural frequency. We will discover that the amplitude of vibration gradually increases to its maximum value of 10 cm at the natural frequency and then decreases at higher frequencies as shown in Figure 4.10. **Resonance** is the condition of large amplitude vibration which occurs when a simple vibrator is driven at a frequency equal to or very near its natural frequency. In addition, the phase of the driving force is one-quarter cycle ahead of the displacement so that the driving force is in phase with the velocity. As we saw in Chapter 3, power is proportional to force and velocity so that our mass-spring vibrator receives maximum power from the driving force when it is in phase with the velocity of the vibrator. When a system resonates, the vibrator achieves its greatest amplitude as the frequency of the driver becomes equal to the natural frequency.

As an example of the principle of resonance, suppose that a bell ringer wishes to ring a heavy church bell, but the bell is too large to be set ringing by a single pull on the rope. However, the ringer pulls the rope as hard as possible and then releases it. The bell then begins to swing, which causes the rope to move up and down. After the bell has performed a complete swing, the rope is back at its original position but it is moving downward. If the ringer now pulls down on the rope again the amplitude of the bell's

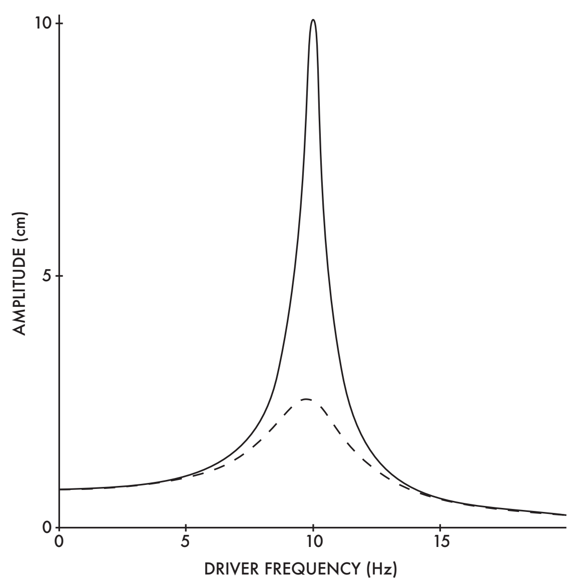

Figure 4.10 *Vibration amplitude versus driving frequency for a mass and spring vibrator with small resistance (solid line) and a vibrator with large resistance (dashed line).*

swing will be increased with only slight additional effort. With repetition of the process the amplitude of the swing increases until the bell is ringing with as much vigor as is desired. The physical force exerted by the ringer is a periodic force with the same period as that of the free oscillations of the bell (and with the correct phase) so the amplitude of the vibration builds to a large value. It is possible to stop the bell by pulling on the rope out of phase with the bell motion.

A struck tuning fork emits a sound having a definite frequency. The sound itself, however, is rather feeble. If the tuning fork is placed on a wooden box with one face open, a much louder sound is heard. In this situation, the driving force (the tuning fork) causes resonance in the box and the large increase of amplitude results in a louder sound (the hollow box is consequently called a resonance box). Suppose now that two identical tuning forks are mounted on resonance boxes with the open end of each box facing the other. Striking one fork will cause the other fork to vibrate by resonance. If the frequency of one fork is now changed slightly by placing a small piece of clay on one prong, very little energy will be transferred by resonance, even though the actual frequency change of the fork has been slight. In this case, the resonance is said to be a **narrow resonance** and is highly selective.

Not all resonators exhibit such narrow resonances. If you blow across the mouth of an empty soda bottle, a sound at the natural frequency of the bottle is produced. If we hold a tuning fork of the same frequency over the bottle, we hear a loud sound due to resonance. If the frequency of the tuning fork is changed slightly by a small lump of clay, very little change in loudness is perceived. Other frequencies close to the resonance frequency also elicit a fairly large response from the bottle. As the frequency is slowly varied from its resonance value, the response of the bottle falls off rather slowly and we have to make a fairly large change in the frequency before the bottle's response decreases appreciably. We describe this situation by saying that the resonance is a **broad resonance**. Some objects, such as the tuning fork on a resonance box, exhibit narrow resonances while other objects, such as the sounding board of a piano, display broad resonances. Most musical instruments have resonance characteristics which are somewhere between these two extremes. Brass instruments have fairly narrow resonances, but they are sufficiently broad so the player can miss the exact resonance frequency by a small amount and still get a good response. If the resonance is too broad, however, the player will not be able to hold the note on pitch.

Why do different vibrating systems exhibit different resonance characteristics? The difference can be attributed to differing amounts of damping (resistance) and sound radiation which remove mechanical energy from the vibrating system. The more rapidly energy is removed the more rapidly the amplitude of a free vibrator decays and the broader its resonance.

The relative amount of resistance in a vibrating system determines whether the system will have narrow resonance or broad resonance. We consider a mass hung on a spring and assume that it is being driven by a small forcing device of variable frequency. If we plot the amplitude of vibration as a function of frequency, we get a response curve like the one shown by the solid line in Figure 4.10. If we now attach a paper umbrella to increase air resistance, friction will have a considerable effect on the amplitude of vibration. Plotting the amplitude for different driving frequencies, as was done before, we get the dashed curve shown in Figure 4.10. Note that with the increased resistance, the originally narrow response curve has been appreciably broadened. Note also that the amplitude achieved at resonance is much smaller for the larger resistance than for the original case. Systems with a narrow resonance respond strongly to a limited, narrow range of frequencies; systems with a broad resonance respond slightly to a greater range of frequencies. Finally, close examination of Figure 4.10 shows that not only does resistance affect the amplitude and thus the width of resonance curves, but it also influences the resonance frequency itself. When little resistance is present the resonance frequency of a driven system occurs very nearly at the natural frequency of the vibrator. As damping is increased, however, the resonance frequency moves to slightly lower values.

We can summarize the effects of resistance as:

(1) A large resistance causes free vibrations to decay rapidly and produces a broad resonance (or response curve) for a forced vibration.

(2) A small resistance results in a long-lasting free vibration and a narrow resonance for a forced vibration.

(3) A narrow resonance has a high amplitude while a broad resonance has a low amplitude.

(4) As the resistance of a driven oscillator is increased, the resonance frequency is lowered somewhat.

4.7 Summary

Simple Harmonic Motion (SHM) is the simplest case of periodic vibrational motion. The period of SHM is the time required to complete one cycle of vibrational motion. The frequency of SHM is the number of cycles of vibration each second and is the reciprocal of the period. In SHM the force causing the vibrator to return to its equilibrium (rest) position is proportional to its displacement. The displacement of a vibrator in SHM can be represented with a sinusoid. "Phase" is the relationship of one point on a wave to a reference point, indicated as a fraction of a period in degrees. The phase difference between two vibrators describes how the motion of one compares to the motion of the other. The natural frequency of a mass and spring vibrator is proportional to the square root of the stiffness-to-mass ratio. When resistance acts on a simple vibrator, the amplitude of the displacement decreases over time. A driving force applied to a vibrator will vibrate at the driving frequency. The amplitude of driven vibration is greatest at resonance, when the driving frequency is the same as the natural frequency. Increasing the damping of a vibrator results in a reduction of amplitude and a broadening of the resonance peak.

References and Further Reading

Crawford, F. S. (1988). "Lowest Modes of a Bottle," *Am. J. Phys.* 56, 329–333.

French, A. P. (1971). *Vibrations and Waves, MIT Introductory Physics Series* (Norton). (See Chapters 3 and 4.)

Hall, D. E. (1987). *Basic Acoustics* (Harper & Row Publishers). (See Chapter 5.)

Rossing, T. D. and N. H. Fletcher (1995). *Principles of Vibration and Sound* (Springer-Verlag). (See Chapter 1.)

Questions

4.1 If a suspended mass is pulled down by stretching the spring, in what direction will it move when released? What provides the force to accelerate it? When the mass reaches the rest position, will it stop or keep moving? What role does inertia play in the behavior of the mass? What forces are acting on the mass when it is in the rest position?

4.2 For SHM, what is the phase difference between the acceleration curve and the velocity curve?

4.3 Consider a mass vibrating on a spring and ignore possible effects of friction. At what position is the kinetic energy of the mass greatest? At what position is the potential energy greatest? How do the sums of the kinetic and potential energies compare at these various points? What happens to the total energy if friction is present?

4.4 Two identical tuning forks are struck identical blows and caused to vibrate. One fork is held in the air, the other has its base pressed against a table. Which tuning fork will sound louder? Why? Which fork will vibrate for a longer time? Why?

4.5 Name some musical instruments which are free vibrators. Name some musical instruments which are forced vibrators. Are the vocal folds free or forced?

4.6 What is the effect of resistance on an oscillatory system? What causes the resistance?

4.7 Make a rough graph of the displacement-versus-time curve for a free vibrator when a small amount of resistance is present.

4.8 List several examples of resonance.

4.9 Will a narrow or a broad resonance allow the greater displacement of a forced oscillator? Which type of resonance is more frequency-selective?

4.10 Describe the mass, the restoring force, and the resistance for each of the following three simple vibrators: (a) a ball on a massless spring, (b) a ball on a stretched massless string, (c) an air-filled bottle with a narrow neck. The relation expressed in equation 4.5, where m is the mass of the vibrator and s is its stiffness, gives the approximate frequency of vibration for these vibrators. An air-filled soft-drink bottle can be viewed as a simple vibrator with the air in the bottle providing the stiffness and the air that moves in the neck of the bottle providing the mass.

4.11 You blow across the mouth of a soft-drink bottle and observe the pitch. You then add some water to the bottle and observe a higher pitch. Has the effective mass changed? Has the effective stiffness changed? (Refer to Question 4.10 for hints.)

4.12 Two identical tuning forks are sounded. When small pieces of clay are attached to one fork it sounds a lower pitch than the other. Has its effective mass changed? Has its effective stiffness changed?

4.13 Explain by using Newton's second law why period increases as mass increases. How can Newton's second law explain the reduction in period which results from increased stiffness?

4.14 Explain how you could utilize resonance phenomena in order to determine the frequency of a tuning fork. If the frequency of the fork were too high, how could it be lowered to the correct frequency?

4.15 Observe and describe natural vibrators such as trampolines, automobiles, beds, floors, and other objects. Identify spring elements, mass elements, and sources of friction.

4.16 Describe examples of resonance for the vibrating systems of Question 4.15.

4.17 Systems like blown soft-drink bottles are often referred to as Helmholtz resonators. Name and describe other systems of this kind. When blowing ceases will the oscillation die out quickly or slowly? What does this suggest about the resonance width (narrow or broad) for these systems?

Exercises

4.1 A mass of 100 gm hung on a spring was set into motion. A movie camera with a framing rate of 20 frames per second was used to photograph the mass. The values for the displacement of the mass obtained by reading each frame of the film are listed in Table 4.2. Determine the velocities from the displacements and compare with the table values.

Table 4.2 *Values of displacement, velocity, and acceleration for the mass and spring vibrator of Exercise 4.11.*

	d(cm)	v(cm/s)	a(cm/s2)
0.00	0.00	6.3	0.0
0.05	0.31	6.0	-12.2
0.10	0.59	5.1	-23.3
0.15	0.81	3.7	-32.0
0.20	0.95	1.95	-37.5
0.25	1.00	0.0	-39.2
0.30	0.95	-1.95	-37.5
0.35	0.81	-3.7	-32.0
0.40	0.59	-5.1	-23.3
0.45	0.31	-6.0	-12.2
0.50	0.00	-6.3	0.0
0.55	-0.31	-6.0	12.2
0.60	-0.59	-5.1	23.3
0.65	-0.81	-3.7	32.0
0.70	-0.95	-1.95	37.5
0.75	-1.00	0.0	39.2
0.80	-0.95	1.95	37.5
0.85	-0.81	3.7	32.0
0.90	-0.59	5.1	23.3
0.95	-0.31	6.0	12.2

4.2 Determine accelerations from velocities in table 4.2. Does a constant force act on the mass? What produces the force?

4.3 Graph the values for displacement, velocity, and acceleration from Table 4.2. (Notice the relationships among these three quantities and that each can be represented with a sinusoid.)

4.4 Construct diagrams representing the displacement, velocity, and acceleration for SHM. If the phase of the displacement sinusoid is 0°, what is the phase of the velocity sinusoid? Of the acceleration sinusoid? What is the phase of acceleration relative to velocity?

4.5 Use the equation from the text to plot the displacements for one cycle of SHM where T = 10 s, A = 2.0 cm, and t = 0, 2, 4, 6, 8, 10s. Use starting phases of 0°, 90°, and 180°.

4.6 The speed of a 1-kg pendulum as it swings through its rest position is 1 m/s. What is its maximum kinetic energy? What is its maximum potential energy?

4.7 A mass of 0.2 kg and corresponding weight of 1.96 N stretches a spring 0.05 m. What is the spring stiffness? What will be the natural frequency of the mass-spring system?

4.8 If the mass of Exercise 4.7 oscillates with a displacement amplitude of 0.1 m, what will its maximum potential energy be? What will its maximum kinetic energy be? What will its maximum speed be?

4.9 The frequency of A4 is 440 Hz. What is its period?

4.10 The period of a tuning fork is 4 ms. What is its frequency?

4.11 When a given simple vibrator is displaced 0.5 cm from its equilibrium position and released, it is observed to vibrate 200 times per second. What is its period?

4.12 When a simple vibrator is displaced 2 cm from its equilibrium position and released, it requires 5 ms to complete each cycle of motion. If a periodic driving force is now applied, at what driving frequency will the vibrator achieve it greatest amplitude?

4.13 What is the amplitude of vibration at resonance for the vibrator with large resistance as shown in Figure 4.10? How does this compare with that of the vibrator with low resistance?

4.14 One way of stating the "width" of a resonance is to take the difference between the two frequencies at which a vibrator has an amplitude equal to one-half its maximum value. Estimate the resonance widths of the two vibrator amplitude curves shown in Figure 4.10.

Activities

4.1 Create a simple vibrator by suspending a mass from a spring. Set the system in oscillation. Measure the period. (You could, for example, time 10 oscillations and this period divided by 10 would be the period for one oscillation.) Give the value of the period in seconds and fractions of a second. Give the value for frequency in Hz. Increase the mass appreciably (e.g., double it). Repeat the above. What happens to the period? Does it increase, decrease, or is it unchanged? What happens to the frequency? Give the value of T and f. Now suspend the original mass on a stiffer spring. Repeat the above. What happens to the period? What happens to the frequency? Give values for T and f. Attach a driving apparatus (your finger, for example) to the mass and spring. Adjust the driving frequency until the maximum amplitude of oscillation is obtained. This is called the resonance condition. Measure T and f under this condition. Are they approximately the same period and frequency as those obtained for the natural oscillation? Why?

4.2 Experiment with a large pendulum such as a mass on the end of a rope or a child in a swing. How do you adjust your pushing frequency relative to the natural frequency to achieve large amplitude? How do you adjust the phase of your pushing force to achieve this result? When the swing is at its position of maximum displacement, the "high point" where it momentarily stops, you exert no force. As the swing begins to move you gradually increase the force (by pushing harder) until you are exerting your maximum effort at the equilibrium position. Your driving force must always be one-quarter of a cycle ahead of the displacement amplitude in order to realize resonance. That is, when the displacement is at a maximum, the force is zero and when the displacement is zero (at equilibrium) the force is at a maximum. Having the right frequency, then, is only part of the resonance story; the driving force must also have the correct phase. If you wish to stop the swing, how do you adjust the frequency and phase of your force?

4.3 Blow on various bottles and observe their resonance frequencies. Identify the "mass" and "stiffness" components of the air in the bottles. Sing into a wash basin while varying your pitch.

4.4 Measure the half-amplitude damping time of a mass and spring vibrator. Add a "tail" to the mass and let the tail move in water or light oil. How does the damping time change? Drive the two systems so that they resonate. Compare their amplitudes.

4.5 Measure displacement, velocity, and acceleration of a moving vibrator by attaching a mass to the end of a suspended spring. Place a Polaroid camera and a stroboscopic light about 1 meter away from the mass. Set the strobe to flashing about 20 times per second. Set the mass in motion. Open the camera shutter for about 2 seconds and pan the camera. Then close the shutter and develop the print. Construct a table having the form of Table 4.1. Fill in the table by measuring the displacements on the print and then calculating the velocities and accelerations. If you do not have a usable print, Figure 4.11 may be employed for these measurements. At which of the tabulated times was the velocity greatest? Smallest? How can you tell? At which of the tabulated times was the acceleration greatest? Smallest?

4.6 Shatter a glass by causing it to resonate. (H. Kruglak and R. Hiltbrand, 1990, "Shattering Glass with Sound Simplified," Phys. Tchr. 28 (Sep), 418.)

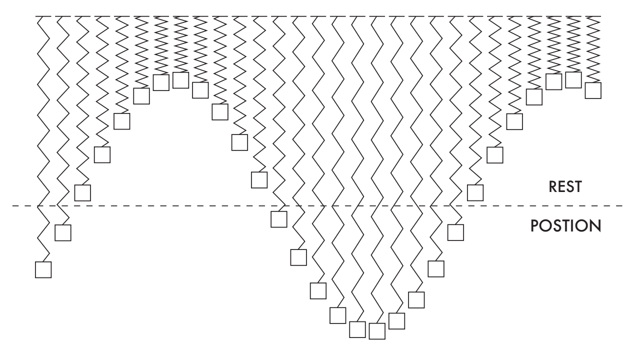

Figure 4.11 *Simulated motion of a mass and spring vibrator.*