.
INCORPORATING SCIENCE INQUIRY CONCEPTS
IN ALGEBRA I LESSON PLANS

by

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TABLE OF CONTENTS

1. INTRODUCTION & BACKGROUND 1

Using Inquiry as a Science Teacher vs as a Math Teacher 2

Challenges in My Classroom 4

Focus of Capstone 7

1. CONCEPTUAL FRAMEWORK 8

Choosing an Engagement 10

Mathematical Modeling versus Modeling Mathematics 12

1. THE LESSON PLANS 14

How the Lessons Fit Together 14

Description of Each Lesson 17

Lesson 1: Translating Words into Expressions and Equations 17

Lesson 2: Translating Word Problems into Algebraic Equations 19

Lesson 3: Rules for Exponents 19

Lesson 4: Multiplying Binomials 20

Lesson 5: Creating and Interpreting Graphs 21

1. PROFESSIONAL REFLECTION 23

REFERENCES CITED 26

APPENDICES 28

APPENDIX A: ALGEBRA I YEAR AT A GLANCE 29

APPENDIX B: LESSON 1 32

APPENDIX C: LESSON 2 44

APPENDIX D: LESSON 3 51

APPENDIX E: LESSON 4 57

APPENDIX F: LESSON 5 64

LIST OF FIGURES

Figure Page

1. Figure 1. BSCS graphic of the 5E model (Adapted from Knowledge *Quest* 9
2. Figure 2. A screenshot of a prompt from the Inquiry Maths website. 11
3. Figure 3. An example of a prompt used in Lesson 3. 20

ABSTRACT

A lack of student engagement and wide range of math aptitude in my Algebra I classes have been big challenges, and direct instruction following the textbook exactly has weakened my joy of teaching. To address this, I created five lesson plans based on a science inquiry style, using 5E lesson format. These follow the existing curriculum and can be implemented next year.

CHAPTER ONE

INTRODUCTION AND BACKGROUND

I teach math at a large, urban public high school. As a high school student, I never would have predicted I would become a teacher. I did not particularly enjoy being a student, as I did not like sitting still and listening to a teacher or following instructions quietly, nor was I motivated by grades.

In my early twenties, I arrived in Philadelphia and started working at a youth program. My role was to teach kids ages 8-18 how to fix bikes and how to ride safely in the city. I rode all over Philadelphia with a single file line of young people pedaling behind me. The kids were so energetic and curious, and I loved it when they exclaimed, “I never thought I could ride so far!” I started to really enjoy bringing new experiences to them and watching them learn. As part of this job, I went to a lot of the public elementary, middle, and high schools in various neighborhoods of Philadelphia. I talked to teachers who lamented about their school’s library being closed for years, about not having supplies or equipment to use in their classes, about the school hiring long term substitutes because they could not find qualified teachers to hire. Multiple teachers told me they often repeated the same lesson all week because so many students were absent each day, causing the few students who actually attended every day to get increasingly bored. Occasionally my job would take me to a wealthy private school, or I would ride through the suburbs and see the new school buildings with abundant musical instruments and science equipment visible in the windows. I was noticing a huge disparity in the educational opportunities available to young people, and I was thinking about how the young people I know in the bike program deserved more. This is why I decided to go back to school and become a teacher.

I went to Temple University and studied physics and education through the TUteach program. The education classes in the TUteach program highlighted a student-centered approach to delivering lessons, rooted in social constructivist ideals. The program puts participants in the field almost right away, doing preservice teaching assignments “to discover first hand if a career in STEM education is a good fit for them, and get a head start in developing teaching skills.” (Temple University College of Science and Technology, n.d.). These placements exposed me to a variety of teaching styles. I had a short time at a Catholic school where the teacher was strict and orderly, students read silently from textbooks and showed aptitude by responding quickly to questions when called upon. A class on differentiated learning placed me at a Quaker school specifically for students with learning disabilities, where the teacher I observed used manipulatives and art to teach math. I did my student teaching at a project-based magnet school where nearly every assignment was a part of a quarter long project, and students rarely had independent work. I graduated from TUteach certified to teach physics and confident that I wanted a to teach in a way that engaged students through questioning rather than telling, and which gave students some autonomy to learn in the ways that best suited them, while building a culture of cooperative learning.

Using Inquiry as a Science Teacher Versus as a Math Teacher

As a physics teacher, I incorporated labs and explorations into my lesson plans regularly. I was part of an active and inspiring professional development network, American Association of Physics Teachers Southeast Pennsylvania Section (SEPS), which hosted workshops for teachers to share lab and lesson ideas at least twice a year. I gleaned many phenomena anchored exploration ideas from the participants in SEPS, especially Bill Berner, an extremely enthusiastic and creative educator who ran the physics demonstration lab at the University of Pennsylvania for 20 years. I attended multiple Modeling trainings through the American Modeling Teachers Association (AMTA) and used the Physics by Inquiry books as the source for much of what I did with my physics students. Another hugely influential resource I used especially in my first years of teaching was a modeling blog, Physics! Blog!, created by Kelly O’Shea (O’Shea, n.d.).

 I enrolled in the MSU MSSE program in 2018 and further developed my understanding of inquiry and modeling approaches to teaching. The particularly influential classes in the program were Engineering Methods for Teachers and Physics by Inquiry, which I still think about when observing the moon. Overall, in the six years that I taught physics, I felt confident and supported in my choice to use inquiry-based lessons, and there was always a plethora of inspiration and ideas to draw from.

Two years ago, my school was having a hard time finding a math teacher. I volunteered to switch from physics to math and have been teaching Algebra 1 to freshmen and geometry to sophomores since then. There are a few key ways that the math department culture differs from my experience in the science department. Since the school is so large, there are about five teachers with sections of the regular Algebra 1 class each year. We are expected to follow a set calendar of curriculum topics to ensure we all cover the same material on time for the common midterms and finals. Another factor influencing the choice of content and pacing is the Keystone exam, a Pennsylvania standardized assessment. Scoring proficient or advanced on the Algebra 1 Keystones is one of the graduation requirements for students in Pennsylvania, and the scores are often referred to when comparing schools in the district, so there is pressure to prepare students well for this test.

The main advice colleagues gave me when I started teaching math was to “follow the textbook exactly.” My colleagues have been immensely supportive as I get used to teaching a new subject, and they are always willing to share resources, but these resources are typically slide sets that use gradual release style examples, or problem sets for students to drill a skill. The gradual release method of teaching is often summed up as “I do-we do- you do.” A typical lesson might start with a definition or rule for students to copy in their notes, followed by an example that the teacher solves on the board while talking through the steps. Students then try an example problem from the board, usually with the whole class moving at the same pace. This is followed by independent practice time, during which the teacher can circulate and assist students who ask for help or who the teacher has already identified as needing more support. There are math teachers who use this style with fantastic results, but in my experience, I find it challenging to meet the needs of diverse learners and I do not see students applying the new skills learned in this way to novel situations. The rote practice of problems helps them only for doing problems almost exactly like what I have demonstrated. In the math department, I have felt less creative when it comes to lesson planning, and I have also found less resources for teaching math through inquiry.

Challenges in my class

There are additional challenges I have noticed since I started teaching math, having to do with students’ readiness for and attitudes about learning math. One factor I suspect is at play is the division of Algebra I into regular and honors levels. After students are accepted to our school, at the end of their eighth-grade year, they take a math placement test. Some freshmen go directly into Algebra II, some are put into Algebra I honors, and the rest are put in regular Algebra I classes. I believe that the students who are most enthusiastic about math, who have had the most effective classes so far, and who place the most value on doing well with math are siphoned into the higher-level classes. The students left in the regular level Algebra I classes often have a sense of low self-efficacy when it comes to math or are coming from schools that did not prepare them for high school level math. Some students continue to need support with pre-Algebra skills, especially involving negative numbers or fractions.

Certainly, there are also some students who have done very well in prior math classes and have a strong sense of numbers and logic, who still end up in the regular level Algebra I classes. The students in my classes represent a wide range of student abilities and prior knowledge. This creates challenges, because some of the students who already learned a concept in middle school become bored, while the students who are seeing something for the first time become confused about why they cannot understand as quickly as the other students.

My school is an academic magnet school, where students have to meet certain academic, behavior, and attendance requirements to attend. The majority of students in Algebra I classes are at grade level according to state standardized tests, but each year there are a handful who are categorized as needing intensive support. These students might have Individualized Education Plans, in which case they have the support of a Special Education Teacher, but often the lowest performing students do not have a documented learning difference and are simply coming to our school with a deep deficit in their prior learning. The problem with the school being a prestigious magnet school is that these students who have learning differences or just have not had the same opportunities as others often develop a crippling sense of not belonging. I need to have lessons which are accessible to these students, which give them a sense of belonging in the class and a way to use their unique skills.

The students with a high aptitude for math who did not elect to take a higher-level course are a special challenge because they are often the type of students who are able to answer questions using logic without writing their process on paper, and they often get annoyed by the emphasis on showing the steps of solving a problem. It is frustrating to me as their teacher to know how capable they are but then have them submit incomplete, disorganized work. I need a way to engage these students with questions that seem interesting and important enough for them to get invested in, so they work at a level that will keep them progressing.

Another challenge I have felt in the past two years has to do with my own enthusiasm for the subject. I consider myself a lover of math; I can sit and do math for hours, losing track of time, and I find this mentally soothing. But teaching math has not been so enjoyable. While I agree with my colleagues that the textbook we use is thorough and it is logically ordered, I have felt uninspired by the dry and teacher centered teaching style I have fallen into. Hearing students lament, “Why are we even doing this?” is one of the most disheartening professional experiences. I miss engaging with students who are curious and excited to share what they know.

For all of these reasons, I would like to intentionally increase inquiry-based lessons in my practice again. I believe this will improve the students’ attitudes and engagement, and their sense of math as a meaningful or useful, and perhaps beautiful, endeavor. Inquiry-based lessons can also reach each individual at the level appropriate for them, which should help me accommodate the wide range of learners in each class. When the teacher is exuding joy about teaching, the learners will notice. Keeping in mind the goal of the math department to offer consistency regardless of who the teacher is, and my desire to collaborate with colleagues, I will use our existing timeline and the department’s choice of textbook, Algebra Structure and Method Book 1, which I refer to as the Dolciani text, as the foundation for my inquiry lesson plans (Brown et al., 2004).

Focus of Capstone

My focus is to develop lessons for Algebra 1 that align with my school’s year at a glance schedule of topics, but which also use inquiry learning. My goal is for students to build interest and confidence in math, while effectively learning the content.

CHAPTER TWO

CONCEPTUAL FRAMEWORK

I am choosing inquiry-based lessons because throughout my courses as an education major in both undergraduate and graduate level, I have read studies that support my notion that the use of inquiry will improve students’ interest and confidence in science, while also ensuring they successfully learn and retain the objectives. I am extending this idea, proposing that inquiry lessons will likewise benefit students of mathematics. While many math teachers are satisfied with their style of delivering content using lecture, followed by repetitive skill practice, the shift in science standards toward more student-centered practices is being mirrored by new math standards as well.

Throughout my coursework I have referred to Douglas Llewellyn′s T*eaching High School Science Through Inquiry and Argumentation.* This text has helped me to understand what makes a lesson an inquiry lesson. For example, simply providing hands on activities does not make a lesson an inquiry lesson. Many science labs are hands on but if the question is provided, the instructions are written, and sometimes even a blank data table is printed, the students are left with little control over their learning and this is not true inquiry. I do not include any lessons in this paper where the students have complete autonomy to propose a question and full responsibility for designing their approach to answering their question, but I am attempting to take concepts that I previously spoon-fed to students using textbook examples and reframe them as something the students interact with, ask questions about, discuss with their peers and build their own understanding of. This matches Llewellyn’s description of the five essential features of scientific inquiry, which are engaging in the question, prioritizing the use of observations, using evidence to make an explanation, evaluating an explanation with new contexts, and communicating or justifying the new explanation (Llewellyn, 2012). In my math lessons, each of these stages are included. Though this looks different in the context of a math lesson compared to what someone may picture in a science lab, the key is that students are taking an active role in their learning.

I used the BSCS 5E Instructional Model, or 5E’s, to write my lesson plans. Each lesson includes these five phases: engagement, exploration, explanation, elaboration, and evaluation (Figure 1). These phases occur in a cycle, and the order can change, or certain phases may occur more than once in a lesson. A historical account of how this model was developed, and the fine qualities of each phase is described in *The BCSC 5E Instructional Model: Origins and Effectiveness* (Bybee, 2006).



Figure 1. BSCS graphic of the 5E model (Adapted from Knowledge *Quest*) (Retrieved June 30, 2024 from https://www.inquirymaths.com).

Choosing an Engagement

The engagement phase of a 5E lesson plan is vital for getting students invested in learning more about the topic. It is valuable because the teacher can gather information about what students already know and what misconceptions they may have. I have tried to choose engagement questions or activities that both give students an opportunity to share what they already know but also challenge them enough that they recognize that they have more to learn.

For most of my lessons I attempted to think of engagements that could apply to students’ non-math lives, or which show how math is a tool for answering broader questions. This is a way to build student interest in the subject, because it demonstrates that math can be used in real life. I try to stay aware of what students consider important, whether that is summer jobs, sports, current events, school clubs and showcases, or even video games, and use these topics in engagements. *For Lesson 2: Translating Word Problems into Algebraic Expressions and Equations Part 2,* I used an opening question about a person who wants to calculate what she is earning at her restaurant job with hourly wages and tips because I know that many of my students with jobs work at restaurants, but also because I know a lot of the freshmen are not working yet but starting to think of getting their first paycheck. To follow up this exercise, the students then think of their own question to answer using the math skills from the lesson, and this can be anything that is important to them. Giving students the chance to insert their own character and interests into a class gets them more invested and also helps me as a teacher to see each student as a unique and whole human, so I try to incorporate this often. However, I am also trying a different sort of engagement in *Lesson 3: Developing and Using Rules for Exponents.* For this lesson, I use a purely mathematical prompt. This is an idea I got from Inquiry Maths (Figure 2).

Inquiry Maths is a website which is a rich source for lesson prompts and other resources, accessible to teachers for free without any commercial connections. The goal, as stated on the site, is to “promote a culture of curiosity, collaboration and openness in the classroom” (Inquiry Maths, n.d.). It was started in 2012 by Dr. Andrew Blair, a teacher in the UK. It continues to grow through contributions of teachers from around the world. Each prompt posted on the site includes a description from the contributing teacher about how the lesson unfolded when they used it in their class, often with examples of student work or examples of other lines of inquiry which might come up. Many of the prompts are thought provoking, and I certainly plan to choose appropriate prompts from this site to use in my Algebra I and geometry classes.



Figure 2. A screenshot of a prompt from the Inquiry Maths website (Retrieved June 30, 2024 from <https://www.inquirymaths.com>).

The prompts on the Inquiry Maths site are “designed to be as devoid of context as possible” (Inquiry Maths, 2024). This is different from the strategy of making math applicable to students’ real lives, but I am interested in using both strategies. Ideally, I hope this demonstrates math is both a useful tool and a beautiful endeavor in and of itself.

Mathematical Modeling vs Modeling Mathematics

I am categorizing each of my five lessons as either mathematical modeling or modeling mathematics. This is a concept discussed in Perspectives on Modeling in School

*Mathematics* (Cirilo et al., 2016). Modeling mathematics takes a mathematical question and approaches it with physical objects or diagrams, or perhaps an interactive computer application, or written symbols to represent the math, but the question is always about math, from the beginning. When the words are reversed, Mathematical Modeling, the meaning shifts quite a bit. In a mathematical modeling lesson, the question is about something outside of the math world, and math is used as a tool for gathering and analyzing the information needed to answer the question. Both types of lessons have value in an Algebra I class, though the mathematical modeling lessons are further from the textbook approach to teaching, and may require more practice for use in my classroom. Words like *authentic* are used to describe a problem presented in a mathematical modeling lesson, but so are the words *messy* and *without a unique answer*.

In lesson 2, students attempt to use a question about how many coins are in a jar. This is a real situation that can be mathematically modeled. However, a count of coins might not be far enough outside of the math realm to truly be considered mathematical modeling. Though there will be a correct answer, there can be a variety of approaches. Lesson 5 is another attempt to include mathematical modeling, by starting with a question about categorizing springs to serve certain purposes, and then using measurements, data tables and graphs as steps in the process of describing different springs. Neither of these lessons is a perfect example of mathematical modeling, but to include a fully open-ended real-world question that requires students to choose what to focus on and try multiple paths towards answering, with no definitive ending point will take building up to.

CHAPTER THREE

THE LESSON PLANS

How the Lessons Fit Together

The five lessons I include in this paper are meant to coincide with the existing lesson schedule used by the Algebra I teachers at my school. This calendar was created by the math department Subject Based Teacher Leader (SBTL) and shared with all of the Algebra I teachers for planning purposes (Appendix A). The goal is for all five teachers of Algebra I to provide courses that are comparable in pace, rigor, and content. As a department, we have limited time to plan together; only a portion of our one-hour weekly meeting is used for sub-department planning sessions. During this limited time, Algebra I teachers are able to discuss and revise the SBTL’s schedule based on how our classes are progressing. We are often at slightly different points of the calendar compared to each other, so we make adjustments in our own classes, deciding to speed up or slow down our pace. There is pressure to cover certain topics by the midterm (early January), Keystone (mid-May) and Final (early June) test times. The midterm and final exams are largely written by the SBTL for all of the math subjects, but as teachers we have some input over what content is included, based on what we have covered by those times. Last school year our final exam did not include any questions about radical expressions because not all of the Algebra I teachers had covered this on time. Unfortunately, we do not have any say about when our students take the standardized Keystone exam. By continuing to follow the existing lesson schedule, my students should be fairly prepared for the common exams, and they should be able to study with peers who have different math teachers.

By following the existing calendar, I am keeping my options open for collaborating with other teachers. If they like an inquiry lesson idea, we can work together to revise it or extend it. Most of my colleagues have many more years of experience teaching math than I do, and they are insightful and passionate about their craft. They are an incredible resource when it comes to predicting student misconceptions or considering realistic pacing for a lesson. If my inquiry lessons were not based on our existing lesson calendar, it would be much less likely anyone would want to collaborate.

Additionally, I actually like the flow of the existing calendar, besides a couple of sticky points. These are three things I would like to address: There is nearly month-long gap between lessons on writing equations and lessons on solving equations. There are a couple of lessons that feel like they are placed into the calendar as afterthoughts, particularly the lessons about negative exponents and scientific notation which are wedged between rational expressions and radicals. There is a sense of repetitiveness when we get to linear functions in mid-January and the book starts by asking which ordered pair satisfies a function. The work involved in these problems is very much like the trial and error used in chapter 1 to choose an answer from a list of answer choices.

The first sticky moment on that list is at the very beginning of the school year. The gap between writing expressions and equations in chapter 1, and actually solving equations in chapter 3, seems frustrating to many students. I certainly do not want students already frustrated in September; that is not a strong start to the school year. In chapter 1, students are focusing on using Algebra to represent a word problem, without having to solve for a numerical answer. Many students want a numerical answer and will skip writing an equation to get an answer as fast as possible. There is often a lot of competition or comparison between students with these initial lessons because some students get answers quickly without following the directions, and other students try hard to follow the instructions but do not know how to get that coveted answer that the person next to them got so effortlessly. In Lesson 1, I am trying to address this issue by including the use of spreadsheets to demonstrate how powerful an equation can be in situations where many calculations of the same type are necessary. My vision is that this will be a satisfying new skill for the students who are able to mentally compute an answer and see writing equations as tedious, and it will also give the students with slower computation skills a chance to learn how to use a tool (the spreadsheet) which can be very useful to them throughout the year.

For the second sticky point about lessons that seem to be randomly wedged into the sequence, without strong connection to the adjacent lesson, I have tried to incorporate some of these into my lesson plans in ways that feel more organic. I include negative exponents with Lesson 3 instead of waiting until April to define these. Scientific notation could be included as an extension in Lesson 4.

For the third sticky point, the sense of repetitiveness at the start of chapter 8, I hope to replace most of the chapter sections of chapter 8 with my lesson 5. This lesson uses a hands-on investigation, where each skill from chapter 8 comes up as natural next step in the process. I want to avoid the repetitive lull of practicing each skill by using context-less drill problems. The graphing skills covered in chapter 8 are typically familiar to most of my students because they made linear graphs in pre-Algebra classes. Having students work in groups for lesson 5 will give the students who remember these skills a chance to hone their skills through helping others, which could alleviate some of the restlessness caused by repeating something they already learned.

Description of Each Lesson

Lesson 1: Translating Words into Expressions and Equations

Lesson 1 coincides with Dolciani sections 1.4 Translating Words into Symbols and 1.5 Translating Sentences into Equations (Appendix B). Lesson 1 falls into the category of modeling mathematics because it does not start with a broader question, it begins and ends within the realm of mathematics. In this lesson students use what they already know about writing expressions, share ideas with each other, and organize their ideas into reference notes they will continue to refer to throughout the year. It is a two-day lesson. Day one focuses on simply writing expressions and equations. In groups of three or four, students work with a set of cards that each have a written phrase, such as “The sum of 8 and x,” of “three more than the product of 4 and x.” I am using phrases directly from the textbook examples, or only slightly modified. Some of the phrases are disconnected from any real-life example, and some of the phrases can be more narrative, such as “I worked 15 hours this week and will get paid (x + 3) dollars per hour. I will be paid \_\_\_\_\_\_ dollars.” The task of the students in their groups is to write an expression to show each phrase using numbers, variables, and operation signs. While doing this, each student keeps a page of notes with examples of phrases that indicate each operation. Each group displays their responses at the front of the classroom for a class discussion in which they assess and explain each other’s responses. Some differences could be errors, while others are equivalent ways of writing the same expression. Students revise their note page based on the discussion then have a chance to self-evaluate by doing an 8-10 question check-in, for which the answer key is provided. This process is repeated with a new set of cards, this time with sentences that can become equations.

During day 2 of the lesson students will put an equation into a spreadsheet to evaluate it for multiple different values of the variables in an efficient way. The goal of including this activity is to demonstrate how equations can be used as tools for repeating a calculation, and to engage the students who have preferred mental math over the meticulous steps of writing an equation.

Since lesson 1 is at the beginning of September, the meta-focus besides learning the math content is for students to practice collaborating in groups, and to establish the class expectation that they are able to provide their own explanation of why something works, rather than expecting the teacher to provide an explanation for them to memorize. Students are talking and sharing in groups, as well as in whole-class discussion. They are moving around between their seats and the front of the room to share their work. They are trying their own rules for answering the check-in questions and scoring their own work. They use the feedback of the check-in to revise their rules. They are getting used to being active learners rather than passive receivers. I do not think it is necessary for the teacher to collect or scrutinize the rules that the students record in their notes. I would read each student’s notes briefly, but in practice these notes are for the student, not for me. If the student is able to use their own rules successfully, that is what really matters, and the teacher will assess this ability with follow up assessments, both formative and summative.

Lesson 2: Translating Word Problems into Algebraic Equations Part 2

Lesson 2 directly follows lesson 1 and presents a question to be approached with mathematical modeling (Appendix C). At the start of the class, there is jar of coins on display and students are asked to estimate how many coins are in the jar. They explain their process, which gives the teacher a sense of their prior knowledge. Then the question is made more complex, “How many pennies and how many nickels are there?” Through guided discussion, the students choose the traits they can quantify about the coins. They build an equation to answer the question. This aligns with sections 1.6 and 1.7 in the Dolciani textbook, where students write an equation relating two variables, and a second equation expressing one variable in terms of the other, then combine the two equations with substitution.

Some students will already have the skills to solve their equations, but some will need to circle back to this after the lessons solving equations. The jars of coins should be saved somewhere safe, without the numbers being revealed, until all students have had the chance to solve their equations. At this point, the coins are counted, and it can be an event to see who came closest to the actual amounts of pennies and nickels.

Lesson 3: Rules for Exponents

This lesson is scheduled for early November and aligns with Dolciani sections 4.1 through 4.4, which cover the rules for simplifying expressions with exponents, and section 7.9 where negative exponents are introduced (Appendix D). The extension to this lesson is an introduction to the next lesson about multiplying polynomials. I have estimated it will take two 50-minute class periods for the students to explore the stations, explain and revise their rules, evaluate and revise again.

The engagement to this lesson is inspired by the prompts on Inquiry Maths website, which are purely mathematical and ask students to make mathematical observations (Figure 3). After discussing their interpretation of the prompts, and establishing how to use exponent notation, the rest of the lesson is heavily inspired by modeling lessons I have used in physics classes. There are stations throughout the room with math statements for the students to consider.



Figure 3. An example of a prompt used in Lesson 3 (Retrieved June 30, 2024 from https://www.inquirymaths.com).

Working in pairs, the students interpret what each statement is saying and how to write it equivalently using variables, or how to evaluate it if they choose values for the variables. They keep a set of notes with their proposed rules for simplifying exponent expressions and are given the opportunity to explain their rules in class discussion, evaluate them with a check-in and to revise as necessary.

Lesson 4: Multiplying Binomials

This lesson directly follows lesson 3, combining the rules the students established for simplifying expressions with exponents with a property they have used before: distribution of multiplication over addition and subtraction. This aligns with Dolciani 4.4 through 4.6. I have planned three days of activities for lesson 4 (Appendix E). The example problems I use in the lesson are similar to the problems in 4.9, about area. This lesson starts with a question about the area of a blanket, which is a physical interpretation of the math skills being covered, but not entirely out of the world of mathematics. I consider this lesson to be a modeling mathematics style lesson, in which students consider variations of a math question and use visual aids to model how to solve the questions. It is inspired by common lessons involving Algebra tiles to teach binomial multiplication, but I have changed the use of tiles to a more generic diagram so the visual can still be used when multiplying any number of terms, with any degrees, using both positive and negative terms.

An extension of this lesson is to have students multiply enough differences of squares, and do enough squaring of a binomials that they are able to identify patterns and explain shortcuts. This will prepare them for factoring trinomials.

Lesson 5: Creating and Interpreting Graphs

This lesson is allotted four days of classes (Appendix F). It aims to cover all of chapter 8 in Dolciani. It is based on a hands-on activity which I have used in my physics classes. To adapt this to an Algebra I class, it focuses more on the math skills involved in creating the graph rather than the physics behind the phenomena. Hooke’s Law states that the amount a spring stretches is proportional to the amount of mass that is hanging on the spring. In this lesson, students make measurements of the springs, record data in a table, graph their data, recognize a linear trend, and write an equation. My goal is that using physical objects such as springs will ensure students relate the mathematical concepts of slope and intercept back to the physical objects they observed. The Hooke’s Law activity in the lesson could easily be switched to a different activity, such as comparing the measured diameters and circumferences of various circular objects (to find pi), or the masses and volumes of different samples of various materials (to find densities).

This is a mathematical modeling lesson, which starts with a question about how to compare different springs to each other so we can choose the right spring for what we want to use it. The math skills are developed as students work through the process of answering this question. I have noticed that chapter 8, about graphing, is a unit that many students already have a lot of experience with; most of the students in my classes will already know how to fill in a data table, plot points on a coordinate plane, and estimate a best fit line. About half of my students will remember how to calculate slope and write an equation in slope-intercept form. The more novel skills introduced in this lesson are being able to find meaning in the slope and intercept on a graph or in an equation, and writing a linear equation in other forms. As an extension of this lesson, additional activities could be included to compare linear graphs to non-linear graphs. For example, after finding the spring constant by the spring length vs mass activity, students could explore how the period of oscillation changes for a spring as the mass changes, which leads to a square root graph. Or if the circumference vs diameter activity is used, students could follow this up with area vs diameter, which will be a parabola.

CHAPTER FOUR

PROFESSIONAL REFLECTION

I did not create an inquiry lesson for every unit on our calendar. Some topics, particularly inequalities, factoring, and radicals, were not easy for me to think of inquiry lesson ideas. Collaborating with colleagues during the school year might lead to inquiry lessons for these topics. I would especially like to get input from Algebra II teachers on ways to teach radicals, as this is a topic introduced in Algebra I but covered in more complexity during the following course. Certain topics may not translate as well to inquiry style delivery, and I might choose to continue to use the textbook examples to give students information. Even if I choose to use the traditional gradual release style of lesson, I can still incorporate more inquiry style questioning and use portions of the 5E lesson, especially having students explain the process in their own words.

Developing 5E lesson plans for math has changed my understanding of the Explain and Evaluate portions of the 5E lesson plan. Previously, I thought of the Explain portion of a lesson as an opportunity for the teacher to give additional information such as definitions or formula, and the Evaluate portion as a graded assessment, something the teacher evaluated to decide if the lesson worked. For the lessons included in this paper, I strived to keep these portions of the lesson student directed. The Explain portions are the students explaining their ideas to each other in group or whole class discussion, or to me through conversation or a collected assignment. Every lesson includes a first round of the Evaluate stage in which students are given a chance to self-evaluate their understanding of the concepts so far. I am using a more cyclical and fluid version of the 5E sections than I have in the past, allowing for students to re-explore and re-explain after their initial self-evaluation. I anticipate I will continue to improve my use of all of the portions of the 5E lesson and my lessons will become more student driven in general.

 I certainly look forward to giving students more opportunities to self-assess their work. Not only will this increase their ownership of their learning, it will potentially decrease the amount of collecting and grading I am doing, freeing me to focus on other aspects of teaching. If I were to do an action research project with data collection and analysis, the use of self-assessments is something I would be interested in researching. My initial thought is that students will take it seriously and will not be tempted to cheat or skip the questions, even though it is not part of their grade. However, I should have a contingency plan if I notice some students are interpreting self-assessment to mean this isn’t getting graded, so I do not really need to do this.

The mathematical modeling idea that students should create their own questions for investigation is exciting to me. I want my students to think of math as a tool to understand non-mathematical situations which they find important. Two of the lessons I include here, Lesson 2 and Lesson 5, have attempts to start outside of math and let students solve a problem by using the math skills. I do not believe either is a perfect example of mathematical modeling, so I would like to develop at least one lesson to use this school year that is more open ended and authentic, which allows students more autonomy in how they use the math. I would like to do this for a lesson in the latter half of the year when students have acquired a varied toolbox full of useful Algebra tools. The unit about two-variable linear inequalities could be a good candidate for the use of a more authentic mathematical modeling task.

The Inquiry Maths philosophy of providing prompts for the students to consider without context is also something I want to experiment more with because I want students to see math as being rich with patterns and surprising connections, and existing in full radiance without the need to connect it to the mundane. Thinking about and doing math can be rewarding and enjoyable, it is not always necessary to use it to solve a real-life problem. I see myself using this style of lesson about once every two weeks, maybe on a set day. Our school allows each department to give tests only on certain days of the week, as a strategy to limit the number of tests an individual student might have to take each day. Since I do not give a test every week, I might add structure and predictability to my class by giving Inquiry Maths style prompts on our test day during the weeks we do not have a test.

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APPENDICES

APPENDIX A

ALGEBRA I YEAR AT A GLANCE

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Approximate dates in the school year | Algebra Topic | Objectives | Chapter in Dolciani | Inquiry Lesson |
| Beginning of September | Introduction to algebra | Translating words into expressions and equations | 1 | 1 |
| Mid-September | Introduction to Algebra continues | Distributive property, reciprocals and division, problem solving | 2 | 2 |
| Beginning of October | One Variable Linear Equations | Solving one, two or multi step equations, transforming literal equations, using fractional coefficients, use equations to solve problems | 3,4.7 (literal equations) | Lesson 2 can be extended |
| mid-October | Rational expressions | Solving proportions/ratios, similar figures, using percentages | 7  |  |
| Late October | Linear inequalities | Solve one or two step inequalities, compound inequalities, absolute value inequalities | 10.1-10.6 |  |
| Early November | exponents | Definitions of exponents, sums of monomials and polynomials,  | 4.1-4.4 | 3 |
| Mid-November | exponents | Multiplying monomials and polynomials, other polynomials, squaring binomials, multiply sum and the difference of two teams. | 4.5-4.6 | 4 |
| Late November | exponents | Area problems | 4.9 | 4 |
|  December | Factoring | Factor natural numbers, greatest common factor in polynomials, factoring quadratics | 5 |  |
| Mid-January-early February | Linear functions | Equations in two variables, plotting points, graphing, slope, different forms of linear equations | 8 | 5 |
| Mid-February | System of equations | Solve by graphing, elimination, substitution. | 9 | Extension of 5 (solve by graphing) |
| Late February | Linear inequalities | Graphing solutions to two variable inequalities, and solving systems of linear inequalities | 10.7-10.8 |  |
| Early March | Quadratic Equations | Solve using zero product property | 5.12-5.13 |  |
| Rest of March and beginning of April | Rational Expressions and equations(Algebraic fractions) | Simplify, multiply and divide, add and subtract, percent problems and mixture problems | 6 |  |
| A couple of days between other lessons |  | Negative exponents, scientific notation | 7.9-7.10 | 3 (negative exponents)4 (scientific notation) |
| Mid-April | radicals | Simplify irrational square roots, simplify square roots of variable expressions, products and quotients, sums, solving radial equations, Pythagorean Theorem | 11.3-11.10 |  |
| Early may | Keystone review | The test is in mid-May |  |  |
| Late May, early June | Review for final exam | Test in Early June |  |  |

APPENDIX B

LESSON 1: TRANSLATING WORDS INTO EXPRESSIONS AND EQUATIONS

|  |
| --- |
| 5E AND INQUIRY LESSON PLAN |
| Lesson 1: Translating Words into Expressions and EquationsUnit Topic: Introduction to AlgebraLength: two 50-minute classesPerformance Expectation: Use observations of a situation or a written description to represent the situation using Algebraic expressions. Distinguish between expressions and equations, and what it means to evaluate versus solve. Use the distributive property to rewrite equivalent expressions.Materials Needed: Notecards with the set of prompts for each group, whiteboards, or easel pad paper. Student Materials: Notecards or other paper to write responses on and tape to the class discussion boards, notebooks or print out of guided notes, their computers to access spreadsheet activity on day 2.Background: This will be one of the first lessons of the year, so in addition to learning math skills, students are practicing how to interact in an inquiry-based classroom. Set clear expectations for how to work as a group and how to participate in full class discussion. This lesson provides practice translating English phrases such as “three more than twice a number” into Algebraic expressions, and phrases such as “4 times a number, minus ten, is 14” into equations. By doing this, students will identify and take note of key words associated with specific operations. Some phrases linked to Algebra operations are already familiar to students, but I have observed that ninth graders often get confused about when to multiply versus when to add, and often have trouble with the order in which subtraction or division expressions should be written. For example, many students take the prompt “five less than four,” to mean $5-4$ rather than $4-5$.A challenge I encountered when teaching lessons using this chapter is that students who are able to mentally compute a solution to an equation do not see value in writing the equation. Given the information “4 times a number, minus ten, is 14,” will say “the number is 6,” and do not see value in writing $4x-10=14$. I want to engage these students right away by showing how powerful a written equation can be for making multiple calculations in an efficient way. The engagement activity is meant to entice these students in particular. |

Engage

 Give a sentence that describes a simple equation, something the students are likely able to do in their heads, such as “If I tell you that the variable *x* is five more than 10, can you tell me what *x* is?” Many students will quickly know *x* must be 15.

 Follow this with a list of as many variations on this same equation that will fit on the board, as shown in the image below. Have students find *x* for each example, but do not give them much time. Ask them how this felt and if they have ideas about how to improve upon the experience.

 

 Show a spreadsheet with a column for each of the numbers in the phrase, and a column for the resulting value of *x.* Type in the values used in the example questions. As they watch, type an equation into the *x* column, then copy that equation for all of the rows. The values of *x* appear almost instantly. Ask the students:

* How this was different from their process? The students should notice that it went much faster (so be mindful to do the demonstration without elaboration or getting sidetracked; it should be noticeably faster.)
* What made this possible? Being able to translate a situation into an equation that can be solved multiple times with different values is a groundbreaking tool.

Of course, this first example is not tied to any real life meaning, so there is not much to gain solving for *x*; a more meaningful example is needed as well.

 Include a second tab in the spreadsheet with an equation representing a realistic situation. There are many options for a real-life example, and this can be tailored to the interests of your students. Since many of my students are interested in getting summer jobs and making money, I might use an example such as this:

“Rosie has a summer job washing dishes at a local restaurant. She is excited to save some money for college and to get some new clothes for school, but she does not entirely trust her manager is paying her correctly, especially because the tips at her restaurant are pooled. She should be getting $10.75 an hour plus one fifteenth of all the tips made that night. She wants to create a spreadsheet to quickly calculate what she should be earning each night.”

Tell the students that they will be able to create a spreadsheet for Rosie after practicing the skills in this lesson.

Explore

 Students work in groups of 3 or 4. Each group is given a set of note cards with written phrases. These phrases are also on the whiteboard(s) or easel paper at the front of the room. The groups’ task is to translate the phrases into expressions. Groups can decide how to collaborate, which could look different in each group. Some groups might give each member two of the notecards to do, then discuss and revise as a group. Other groups might look at each notecard all together, one notecard at a time. As the groups are working, circulate in the room to make sure all students are participating. A set of written phrases to use on the cards to pass out is included below, followed by an answer key.

|  |  |
| --- | --- |
| The sum of x and 10 | The product of n and 5 |
| 17 more than b | One half of a number |
| d more than 3 | The quotient of x and 8 |
| The difference between 20 and a number | 9 divided by z |
| The difference between a number and 20 | One third of r |
| X decreased by 15 | 7 decreased by 8 times a number |
| 7 minus y | 7 decreased by 8, times a number |
| Twice a number, added to 30 | Twice the sum of a number and 30 |
| Six times the difference of 8 and a number | Three increased by five times a number |
| 3 more than two thirds of a number | The sum of 6 and x, divided by 8 |

|  |  |
| --- | --- |
| The sum of x and 10x+10 | The product of n and 55n |
| 17 more than b17+b | One half of a number½ x |
| d more than 33+d | The quotient of x and 8x/8 |
| The difference between 20 and a number20-n | 9 divided by z9/z |
| The difference between a number and 2020-n | One third of r1/3 r  |
| X decreased by 15\x-15 | 7 decreased by 8 times a number7-8x |
| 7 minus y7-y | 7 decreased by 8, times a number(7-8)x |
| Twice a number, added to 302x +30 | Twice the sum of a number and 302(x+30) |
| Six times the difference of 8 and a number6(8-x) | Three increased by five times a number3+5x |
| 3 more than two thirds of a number3+2/3 x | The sum of 6 and x, divided by 8(6+x)/8 |

When a group agrees on an expression to represent each phrase, they write each expression on a separate piece of paper. They should write large enough that the expression can be read by the class when it is displayed at the front of the room, and they should include a way to identify themselves such as writing their group number on the back of the paper. One student from each group pins or tapes the papers on the corresponding whiteboards at the front of the room. When all groups’ responses are displayed, lead a class discussion about each whiteboard.

Start the discussion with broad prompts, such as “What are some things you notice about the response taped on this board?” If students are not volunteering observations, or if there are only a few students participating, encourage more students to participate by offering more specific prompts, such as:

* Do any groups have the same thing written on their papers?
* Are any of these expressions written differently but mean the same thing?
* Are there any expressions that do not match the others on this board?
* Look at the expressions on this board, are they similar to any expressions on other boards?

During the discussion, students make notes about what word phrases indicate each of the four basic operations: addition, subtraction, multiplication and division. Depending on the students’ notetaking skills, you can provide a printed copy of a table such as the one shown below to scaffold this effort. It is not necessary during this stage of the discussion that the specifics about what phrases go with what Algebra operations be agreed upon by the class. Instead, let students make their own preliminary notes.

|  |  |  |  |
| --- | --- | --- | --- |
| English phrase | Record the parts of the phrase that indicate a specific operation | How would you write this phrase algebraically? | If there an equivalent way to write the expression, show it here: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Example of how the students might be taking notes during the class discussion.

|  |  |  |
| --- | --- | --- |
| operation | Phrase | Examples of translation |
| Addition |  |  |
| Subtraction |  |  |
| Multiplication |  |  |
| Division |  |  |

Example of how the students might organize their final notes about phrases that indicate each operation.

Explain

 In their groups, students compare notes and explain any differences to each other. For example, if one member of the group translated “4 less than x” as “x-4” but all the other students recorded “4-x,” the students can each explain to each other why they chose the order they did, with the goal of coming to an agreement about which is correct. Another example of a difference students might discuss in their groups is whether or not two expressions are equivalent, such as “3x” and “x3.” The phrases that involve grouping symbols may need the most discussion. Compare “the product of 4 and *x*, plus 7” to “the product of 4 and the sum of *x* and 7.”

The fist phrase is $4x+7$, and the second phrase is $4(x+7)$. Students will have to pay close attention to how the words are grouped so they correctly use grouping in their expression.

Evaluate

 Give students an 8-10 question check in at the end of the class period, which they do with the help of their note page. Questions from Dolciani chapter 1.4, page 16-17 can be used. Show the answers so students can self-check. They should circle any questions they answered incorrectly. Give groups time to assist each other in correcting errors, perhaps re-explaining to each other or perhaps revising their notes.

 Give students a second, similar check in the following day. Collect the second assessment and make note if there are common errors indicating if there are phrases or operations for which the students need additional examples.

Repeat the Explore-Explain-Evaluate phases of the lesson, using cards that have written sentences to translate into equations. After passing out the new set of cards, ask the students what a key difference is between the card sets. The students should notice all of the new cards make statements that compare two values. The new cards give two expressions, connected by an equal sign. Define equation and expression for the class. A key word that indicates where an equal sign belongs is “is.”

Extend

 Students will end the lesson with a reference table of phrases and notes about how to write each phrase algebraically. They will keep this in their notebooks and can refer to then in future lessons. They might find more information to add to this table as they learn more, especially as they learn other equivalent ways to represent an expression, such as x/3 being equivalent to ⅓ x.

Day 2:

Engage

Start the second day by referring back to the example with Rosie calculating how much she should be paid each day. Ask students to identify three situations in their life where they might want to use an equation. If the technology is available, students could write their ideas on a shared electronic whiteboard.

Explore

 Give the students a page of values, written as if it were Rosie’s notes. The spreadsheet about Rosie should be shared with the class so each student has an editable copy. In groups, students decide what equation will work for Rosie to calculate her pay each day, and what information she will need to plug into the equation each day to do the calculation. The column headers should identify the values being used in the calculation, for example “number of hours worked that night,” and “total amount of tips made that night.” The rows should be labeled with the date. Let students work in groups to design the spreadsheets but be able to assist any groups that are stuck. Students answer any follow up questions included in the activity and submit their individual work online.

|  |
| --- |
| Help! I thought I would have the money I wanted for getting school supplies saved by now, and I really want to get a ticket for the Roots Picnic, but I do not know if I can! I worked for a whole week straight and thought I would have so much money by now, but I really didn’t save much. I am starting to wonder if I am getting paid fairly. I wasn’t really paying attention because I trusted Kelly the manager, but now I am starting to wonder! My hourly rate is $10.75, which I thought was pretty good, plus they told me I would get my share of all the tips each night, which would be one fifteenth!On Monday, I worked 6 hours, and I heard Casandra tell Amir that we had a total of $568 in tips that night! That sounds like a lot to me, I am not sure I was paid enough.But then on Tuesday I only worked 4 and a half hours, and the tips were real weak, only $133. Wednesday, Thursday, and Saturday I worked 5.5 hours each day, but Friday I was working 7 hours!! We made the most tips on Friday of course: $680. I asked Casandra if that was normal, and she said only for Fridays. Wednesday tips were $200 exactly, and Thursday and Saturday the tips were $389, then $471. How much should I have been paid by now? |



Answer key for Rosie’s paycheck question.

Explain

After completing the activity with the fictional scenario (Rosie’s job), students write their own question to calculate with an equation. This could be done individually, or in the groups depending on how many students were able to think of real-life examples that could be translated into interesting yet manageable equations. Read the student list before this portion of the lesson and be prepared to guide students or groups to choosing situations that will work well for the activity. Students then create and explain their own equations, with multiple iterations being solved in a spreadsheet.

Extend

 A limitation to this activity is that students have not yet been taught how to rearrange equations to isolate variables. This limits their real-life examples to creating equations where the unknown is already isolated. This activity could be revisited after lessons on rearranging equations, and it may be possible to calculate more of the student’s proposals.

APPENDIX C

LESSON 2: TRANSLATING WORD PROBLEMS INTO ALGEBRAIC EQUATIONS PART 2

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| --- |
| 5E AND INQUIRY LESSON PLAN |
| Unit Topic: Translating Word Problems into Algebraic Equations Part 2Length: 50 minutesPerformance Expectation: Describe a situation in words and translate this to values, variables, and operations. Clearly define the variables. Combine expressions into a single variable equation.Materials Needed: Sealed jar(s) of mixed coins, digital balance or balances capable of precisely measuring the small mass of single coins and also the total mass of the full jar. Loose coins in addition to what is in the jar and another identical jar that is empty. One jar should have a mix of pennies and nickels, other combinations could be dimes and quarters, or quarters and half dollars.Student Materials: Whiteboards and markersBackground: Guessing the number of items in a jar is a popular contest, but how does this get more interesting when there is a mix of items, and you want to determine the number of each? The jars of mixed coins can also represent other mixtures, such as geological or atmospheric samples, body composition, or chemical compounds.This lesson is meant to come after lessons about writing expressions and equations with only one variable. Some students might have experience solving an equation from prior classes, but at this point of the current the school year, we have not isolated variables. The opening question about the number of coins in the jar can be revisited about later, after lessons about the using inverse operations and the properties of equality. |

Engage

 Opening: At the start of class students will be shown a closed glass jar that has a mix of nickels and pennies, and the teacher poses the questions, “Without opening the jar, what are your strategies for determining the number of coins? How many coins are in the jar?” Give the students a few minutes to write down their own responses, then a few minutes to discuss with a partner.

Share prior knowledge: Listen to partners talk about their plans and select a couple of students to share ideas with the class. It is likely that students will have ways to estimate the coins based on how many they can count at the surface of the jar, or by considering how many coins they can hold in their hands compared to how many handfuls of coins it might take to fill the jar. As a class, discuss what mathematical concepts the students are using; it is likely they are focusing on volume and proportions. Ask the students how they could accurately predict the number of pennies and the number of nickels. Give students a minute or two to discuss ideas with their partner.

Explore

Give students more information about the jar of coins which they can use to develop their plans. If their initial ideas focused on the volume or shapes of the coins, encourage them to brainstorm what other measurable properties are consistent for each type of coin. Introduce the balance(s) and test with the class if all nickels have the same mass, and if all pennies have the same mass. Students should record these masses in their notes as “things we know about the situation.” Ask the class what else we could measure with the balance. They should suggest the full jar, but if they do not also want to measure the empty jar ask them if the measurement of the jar full of coins is the mass of the nickels and the pennies only. Give the pairs time to record what they know about the situation. At this point, if they have not already stated this, someone in the class will want to measure the empty jar to subtract that mass from the mass of the full jar. Have the students define variables that can be used with the bits of information that they have to write an equation. If they write an equation such as p + n =total mass of coins but state that *p= number of pennies* and *n= number of nickels, give* an example with the loose coins to show that the numbers of the coins do not add up to the measured mass. What do we have to add to get the total mass? The mass of each coin, multiplied by the number of that type of coin.

|  |  |
| --- | --- |
| Coin | Mass of one coin (g) |
| penny | 2.5 |
| nickel | 5.0 |
| penny | 2.268 |
| nickel | 5.670 |
| Half dollar | 11.340 |

Let students use balances to measure individual coins, and perhaps combinations of coins. The mass of two pennies is equivalent to the mass of one nickel. If *p* represents the number of pennies, then the total mass of pennies is 2.5*p.* There is a similar expression for the total mass of nickels, 5.0*n.*

When the students have correct (but possibly different) equations to show that the mass of all the pennies added with the mass of all the nickels is equal to the total mass, ask if they can solve their equations. They cannot because there are still two variables in the equations. They will need to know more about how the pennies and nickels compare to each other. “We need to express one of the variables in terms of the other.” This is new vocabulary, so spend time giving examples of what this means.

What other information would make this solvable? We need to know how the variables *p* and *n* compare to each other, and since these variables represent the number of each type of coin, we need to know how the number of pennies compare to the number of nickels. What are some possible ways this information might look? We might know there are equal numbers of pennies and nickels, *p=n,* or that there are two pennies for 3 dimes, 2*p*= 3*d*, (or any other proportion), or we could know the total number of coins, *p + n = total number of coins.* Give students another piece of information such as these about the coins in the jar. Make sure to use true information, so the problem is actually solvable.

After students have two correct equations for their two variables, and they have substituted to eliminate one of the variables, they should write their answers to the original question, “How many of each type of coin are in this jar?” as complete sentences.

Explain

Each student should write the steps they took to solve this problem in their notes. Instruct them to focus on the steps for writing the final equation, without worrying too much about the physical steps such as how to use the balance. This can be done as a closure for the first class period, and if there is time have a student volunteer to talk through the steps for writing the final equation.

These are the steps:

1. List what is known about the situation
2. Clearly identify the variables
3. Write an equation using what is known
4. Write a second equation that expresses the relationship between the two variables
5. Substitute the second equation into the first so there is only one variable left in the equation
6. Take the steps to solve for the variable

Explain

Students should be able to use the same process as recorded in their notes to solve similar yet novel problems with two variables. Start with a similar problem and build to more variations. As the students to these problems, they are using their written explanation about the steps required. While using their own written steps to attempt more problems, they are checking and revising their explanations.

As a similar problem for students to check their steps, present them with a second sealed jar with a combination of a different type of coin. Have students determine the mass of the coins and record the values of mass for the individual coins. Tell students a second piece of information, such as “there are three times as many quarters as there are half dollars in this jar.” Instead of just trying to get the answer, remind students that they are following the steps they recorded in the first part of the activity. Their goal is not just to answer the question, but to evaluate how successfully they have explained the process. If they are unable to follow their own explanation, they will need to revise the steps.

Evaluate

 The students should be able to use their process to write equations for the questions in the textbook. Students should also be able to come up with their own questions to translate into equations.

Extend

 The coin jar question can be revisited after lessons about solving equations. The students can then solve the equations they wrote and produce answers for how many of each type of coin are in the jar, and the coins can be counted to see how close the students’ answers are to the accurate numbers. This activity can be referenced again when students are learning about percent difference and percent error during the percentages and ratios lesson later in the year.

 The textbook includes many questions that seem like riddles, more than actually important questions with realistic bits of information. For example, the first question in the problem set is “An oil painting is 16 years older than a watercolor by the same artist. The oil painting is also three times older than the watercolor. How old is each? Choices for the watercolor’s age: 4, 8, 12” gives information that does not make sense for someone to know without already knowing the ages of the paintings. It is asking a question by giving information that we would only realistically have if we already knew the answer to the question being asked. It is frustratingly circular to some students, even if others accept this type of question as a task to do without wondering why they are doing it. This type of riddle question makes it easy to give sufficient practice with appropriate levels of difficulty, especially when just learning a skill. But to keep students engaged and to give examples of how this math skill is applicable in real life, there should be references to a wide variety of real-life questions to write equations for. Topics to create problems of this type could include problems in a variety of fields such as healthcare, personal finance, city planning, geology.

APPENDIX D

LESSON 3: DEVELOPING AND USING RULES FOR EXPONENTS

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| --- |
| 5E AND INQUIRY LESSON PLAN |
| Unit Topic: Developing and using rules for exponentsLength: Two 50-minute classesPerformance Expectation: Understand and use exponent notation; be able to represent terms with exponents in equivalent ways. Devise, check and revise a set of rules to use when performing operations with exponents. Background: Instead of delivering the rules involving exponents as a lecture or as notes for students to copy and memorize, this unit plan has students investigate various prompts at stations which will guide them to making their own rules. They will then test these rules to make sure that when used with numbers, the results are correct. Students will create their individualized notes based on their observations and use of logic while interacting with the prompts, they will also take special note of what surprises them or is potentially confusing to them. Ultimately students will have these exponent rules in their Algebra toolboxes by the end of the lesson:* The meaning of an exponent (and how this is different from multiplying)
* The value of any number raised to the exponent zero or any number raised to the exponent 1
* The meaning of negative exponents
* Two terms can be added only if the base and the exponent are the same
* Product rule for exponents
* Exponent outside of a parenthesis expression with only multiplication or division can be “distributed”
* Power of Power rule
* Division rule

The engagement activity is inspired by the Inquiry Maths website. It is simply a display of two mathematical phrases without any instructions. I let students know that I would like to see their responses written on paper. Most students in my Algebra 1 sections know how to raise a number to an exponent, but there will be few who are unfamiliar. I anticipate that a couple of students in each section will incorrectly state that $2^{3}$= $2∙3$. |

Engage

Display the two phrases,$ 2^{3}$ and $2∙3$, on the board. Tell students to observe the expressions and write statements about what they notice or wonder. Some reactions I anticipate are that students state these are not equivalent phrases, that they numerically evaluate each, and perhaps that they express each as equivalent expression. This is a quick engagement, only a few minutes. A scan of the student’s notes will let me know if any of them do not yet know what it means to raise a number to an exponent. If there are students who are not writing anything, ask a question to get them started, such as “are these the same?”

Have students discuss what they wrote with the person near to them, and as they do so listen and choose a pair to share out to the class.



Example of how a student might respond in writing to the prompt.

Explore

Use the expression from the lesson opener, $2^{3}$, to establish some vocabulary and techniques for testing and generalizing an assumption.

* The base value in this example is the 2, and the exponent is the three
* There are a few ways this is said out loud, “two to the third power,” “two cubed,” “two raised to the third,” are some examples.
* To generalize the phrase, replace the numbers with variables. $2^{3}$ becomes $x^{n}$, and $2^{3}$ = $2∙2∙2$ can become $x^{n}=x∙x∙x…$ *n* times.
* Show one of the prompts that the students will soon look at, $(x^{n})^{m}$, and show them they can test the rule they come up with by choosing numbers to replace the variables with. For example, change $(x^{n})^{m}$to, $(4^{3})^{2}$, or $(5^{2})^{3}$, or any other set of values in the same format, and evaluate by using order of operations. (Do an example of this evaluation as a class.) They now have a value to compare to what they get when using the same values for each variable but applying their rule to simplify instead of using order of operations. Calculators can be used for this part if you choose to allow this!

Give students instructions about how to do the exploration activity. There are four different stations with prompts similar to the engagement prompt. Some prompts will have additional instructions, but the basic goal is to come up with rules to simplify the indicated operations involving exponents. Students will record notes and preliminary rules on a separate page for each station, to leave plenty of space for further testing and revising for each type of expression.

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Print these prompts for the stations.

The stations can be visited in any order. After everyone has made it to all four stations, give student time to discuss amongst themselves. At this point, their rules will probably be statements about what works, but it is acceptable to include rules about what does not work as well. It is also acceptable to include questions. They may find what they agree about, and they may find differences. Encourage students to plug in values to the expressions and evaluate to show their rule works. The homework is to list the revise their rules and rate how confident they are with each, identifying the ones they are most unsure about. I anticipate this exploration, plus time to compare notes with classmates, will take about 30 minutes.

To support students with less familiarity with exponents, have printed tables of values of numbers raised to exponents. Calculators can also be available for students who would be unnecessarily burdened by multiplying the larger numbers they get when substituting values into the exponent expressions.

 Explain

At the start of the next class session, have the prompts from each station displayed on whiteboards or easel paper at the front of the classroom. Give students ten minutes to write a rule they are confident about and tape it on the corresponding board. They may post more than one if they like, but each student should do at least one. Look at the results of this share out and discuss briefly as a class. Are there any prompts which have no rules? Could anyone contribute something, even a question or a rule about what does not work, to these prompts? Are there any prompts that have the same or similar rules posted by multiple students?

Select a prompt which students seem to be confident about, and for which their rules are correct, and have a volunteer explain the rule they posted. Select a prompt that has fewer or no contributions and as a class choose numerical values to plug into the expression. Then evaluate and ask the class if they see a pattern that suggests a possible rule. Try plugging in different numbers to test the possible rule.

Give students time to revise rules in their groups.

Evaluate

Students will self-evaluate their rules by using them to do a problem set.

Give them ten minutes to do the problems, then ten minutes to check their answers and identity where their errors are coming from. Did they misuse their rule, but are still confident the rule is correct? Do they therefore just need more practice using the rule? Or is the rule incorrect?

Extend

When the basic rules are set, follow a similar but possibly abbreviated process for developing rules for expressions that include coefficients and distribution. The prompt with the binomial being squared will be the topic of a future lesson but give students the opportunity to start considering it now.

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Print these prompts for extension activity.

APPENDIX E

LESSON 4: MULTIPLYING BINOMIALS

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| 5E AND INQUIRY LESSON PLAN |
| Unit Topic: Multiplying BinomialsLength: three 50-minute classesPerformance Expectation: Use diagrams and equations to show how to arrive at the simplified form of the product of a monomial and a binomial or trinomial, two binomials, or a binomial and a trinomial. Refer to the rules of exponents to show how to square a binomial. Notice patterns such as the characteristics of the polynomials that result from multiplying a sum and a difference, or from squaring a binomial. Students will be able to extend their understanding to setting up multiplication of other degrees of polynomials.Background: In the previous lesson about rules for simplifying operations involving exponents, students were able to multiple monomials such as $\left(3x^{2}\right)\left(4x^{5}\right) $by using the associative and commutative properties of multiplication along with the product of powers rule. In this lesson, students will use the distribution of multiplication over addition or subtraction to multiply binomials, trinomials and other polynomials.Most of the practice problems in our textbook that have to do with multiplying binomials are questions about the area of rectangles. For example, a problem might ask students to write an expression for the area of a rectangular garden that has a length which is 4 m longer than twice its width. This leads to:$$width=w$$$$length=2w+4$$$$A=lw$$$$A=\left(2w+4\right)w$$$$A=2w^{2}+4w$$Another example, leading to multiplication of two binomials is “A rectangular horse pen is changed in size such that the width has 7m added to it, and the length which was originally three times the length is now decreased by 2 m. Write an equation for the new area.”$$new width=w+7$$$$length=3w-2$$$$A=lw$$$$A=\left(3w-2\right)(w+7)$$$$A=3w\left(w+7\right)-2(w+7)$$$$A=3w^{2}+21w-2w-14$$$$A=3w^{2}+19w-14$$Questions like these can be nicely represented as a diagram or with Algebra tiles. There are many sources of examples and instructions on how to show binomial multiplication using Algebra tiles. A clear and concise video which I recommend is <https://www.youtube.com/watch?v=mNm-FwTD0pQ> In most explanations of how to use Algebra tiles, the final product of the binomials is found by counting the tiles and adding the like tiles: the $x^{2}$ tiles, the $x$ tiles and the *1* tiles. This certainly leads to a correct answer, but I believe it is missing the greater idea that each term in the binomial is being distributed to each term in the other binomial. Without understanding binomial multiplication as a special case of distribution, there is no way to expand upon the skill for multiplying higher degree polynomials. Likewise, if students are simply taught to F.O.I.L. they will not have the instructions for multiplying a binomial by a trinomial, or other higher degree polynomials.For this reason, I am modifying Algebra tile lessons slightly to emphasize the underlying property being used: distribution. Instead of using fixed sized blocks and counting individual pieces, the students set up diagrams and show the multiplication of each pair of terms in the segments.The first example from above would look like this:And the second example would look like:In these diagrams, the scale is not very important. It is not necessary that the students carefully show the section labeled 2w as being twice as long as the section labeled w. And they do not need to know how the variable, w, compares to the constant 4. Please note that the actual value of 2w might be less than 4 even though the diagram shows the segment 2w being longer than the segment 4. Infect, it is fine to show all segments as equal lengths. Let students know this. Ultimately a diagram to show the distribution of each term in a higher order polynomial can be used in the same way. For example:This visual is not about showing an area of something like a garden or a horse pen, it about showing that both terms in the binomial x+2 are being multiplied with each of the three terms of the trinomial $x^{2}+3x-8$ |

Engage

Start with a simple question to remind students of something they will likely know how to do, finding the area of a rectangle. Start without variables, for example:

I am knitting a blanket for my new niece. So far it is 32 inches across and only a few inches long, so I have a way to go! I want it to be 32 inches by 44 inches. What will the final area be?

Have students draw a diagram and label each side with the dimensions, and walk around the class to be sure they all know the area of rectangle can be calculated as length times width. Show a couple of ways to write the unit for this area: sq. in., or $in^{2}$.

Now tell them “I decided the blanket is not large enough. I know I want to extend the length, but I am not sure by how much, so I will just say I am extending the length by *x* inches for now. Draw a new diagram to show this. Can you write an expression for the area still (it will have the variable *x* in the expression)?

Circulate in the room to make sure students are finding an area using the unknown variable. Have a volunteer show their process at the board. Ask the class how we could write this problem without using the diagram:

$$A=32(44+x)$$

And what property they will need to use to simplify the expression (distribution.)

$$A=32∙44+32x$$

Give the students a new area problem, this time with variables in both the length and the width:

I am also knitting a blanket for my sister. I’m not exactly sure what the dimensions her blanket will be when I am done, but I am starting with a piece I already made that is 20in by 30in. I am planning on extending the width by the same amount that I extend the length.

Work with your partner to draw a diagram, then determine an expression for the final area of this blanket. (Your expression with still have the variable x in it.)

Explain

Explain to the students that these diagrams are a visual aid for them to reference when explaining the mathematical steps they are taking to multiply the binomials.

Ask the students what expressions for the length and the width of the blanket are.

$$width=32+x$$

$$length=44+x$$

These are both binomials; we are multiplying two binomials when we do this problem. You just did a good job multiplying two binomials by using your diagram, but your task now is to come up with a solid explanation of how to do so without having to draw a diagram. Use this last question to write the steps you showed in the diagram but without the diagram. Notate each line of your work with the Algebra property you are using.

Students work together to do this, with prompting as necessary.

Evaluate

Day 2: When they have established the steps, give them a short check in to evaluate their ability to multiply binomials. They may set up diagrams or show the steps line by line with distribution. Show the answers so students can decide if they need more explanation, more practice, or if they are ready to move on.

Extend

Day 3: As an extension, have students find patterns by doing a number of problems in which they multiply a sum times a difference (difference of squares) and a number of problems in which they square binomial. Discuss with students as they find the pattern how this pattern can be useful as a shortcut.

As another extension, present problems with polynomials that have more terms and have students follow their same steps of breaking the first polynomial into terms and distributing each term over the second polynomial. In theory, they can do this with any number of terms.

APPENDIX F

LESSON 5: CREATING AND INTERPRETTING GRAPHS

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| Unit Topic: Creating and Interpreting GraphsLength: four 50-minute classes* One class for introduction, setting up tables and collecting first data set
* Second class for collecting another set of data and setting up graphs
* Day 3: Share and discuss graphs and answer analysis questions
* Day 4: Extension problems from textbook and other sources

Performance Expectation: Students collect data and record it in a data table. Students set up graph axis based on domain and range of data points, with clear labels about what variable is on each axis. Students plot data points and add a best fit line by estimation. They refer to the physical situation to explain if this is a discrete or continuous graph, and what the boundary conditions are. They find the slope of the best fit line, the y-intercept, and write the equation for the line in slope intercept form. They identify the physical meaning of the slope and the y-intercept. They use their graph to make predictions.  Materials needed: Hooke’s Law spring set (enough for each group of three to four students to use one spring at a time, and then switch to test with a different spring), a variety of hooked masses, metric ruler or meter stick for each group, graph paper.Teacher note: Collect the data for all of the springs ahead of time and plot using your preferred graphing program. Label the springs (#1, #2, etc.) and keep a list of which springs have which spring constant. This will enable you to spot errors in student’s data quicker, which will allow you to guide the groups to the extent you deem necessary to ensure they are successful.Background: This lesson is based on a lab I used in my physics classes for the study of spring force and spring energy. There are many other variations of simple data collection activities that lead to linear graphs. Such as measuring the diameter and circumference of various circular objects and plotting circumference vs diameter to find the slope is pi, measuring position of an object which moves with constant velocity and plotting position vs time to find the slope is the speed the object moves at, or measuring the temperature of a pot of water on a constantly heating hotplate and plotting temperature vs time, then comparing to other solutions being heated in the same way.For good data in this lab, the springs have to be particularly consistent in how they stretch and have low enough spring constants that their stretch is measurable with changes of mass in about 50-100g increments. You should have a variety of different springs so the students can observe that the slope for the length vs mass graph changes depending on how stretch or not stretch the spring is. It is possible to buy sets of springs specifically for science experiments, called Hooke’s Law Springs.  I have the equipment because I taught physics; I recommend checking with your science department to see if they have the springs and spring stands required. |

Engage

 Opening video: “How do astronauts weigh themselves in space?” <https://www.youtube.com/watch?v=oU3pp_4n84U&ab_channel=CanadianSpaceAgency>

Over the years I have found various videos of astronauts using the spring oscillator to measure their mass on the International Space Station. Note that in the video mentioned above, the Canadian astronaut, David Saint-Jacques, refers to the spring as a swing.

Before showing the video ask the students leading questions:

* What would I use if I needed to weigh myself? (a bathroom scale)
* Are there other, perhaps creative, possibilities that you can think of? (a balance beam with a known weight on the other side, seeing how much I make a floating container sink, have a strong person lift you and compare the effort to lifting other weights)
* If I were an astronaut on the International Space Station (ISS), would any of these methods work? (Let students discuss with a partner, they should recognize that with the very low levels of gravity on the ISS, most of the methods they mention will not work. If they are not familiar with the weightlessness of astronauts on the ISS, show them a snippet of Sally Williams giving a tour of the station: <https://www.youtube.com/watch?v=XkM_04Ch76E&ab_channel=VideoFromSpace)>

In the video Saint-Jacques shows how he uses an oscillating spring to measure his mass and compares how the spring moves with him on it to how it moves with nothing on it. He does not talk about spring constant or explain the physics of why the spring acts like this. I like this video for building curiosity without giving answers.

Explore

Show an example just like in the video of Saint-Jacques, but use a spring from the lab supplies and a 50g mass. Tell the students the mass is 50g. Give students time to observe and take mental notes of what they see, then change the mass to 100g and let the spring oscillate again. Tell the students the mass is 100g. Ask students what they notice.

* The spring appeared to move faster when it had 50g on it, then slowed when the mass was doubled.
* The spring was stretching lower when the mass was greater.

Put a mystery mass on the spring. Use a small bag with a laboratory mass or other object inside so the students are not answering questions about it based on its appearance. Put it on the spring and ask students what they can tell you about the mass. The students should observe how the spring oscillates compared to how it oscillated with the 50g and 100g masses. If the spring is oscillating quicker than in the first demo, they will know the mass is less than 50g. If it is moving slower than it did in the second demo, they will conclude the mass is greater than 100g. You could use a mass in between 50g and 100g, but this difference in oscillation will be challenging to recognize using casual observation.

Give students a moment to record how they could use a spring to determine the mass of an unknown object.

Next, have three or more springs with different spring constants set up at the front of the class. Hang the same amount of mass on each, and let them oscillate. A good way to start the springs oscillating without adding extra variables is to hold the mass up so the spring is only slightly stretched, then gently release it. Students write down their observations of the springs in their notes.

The students should notice that the three springs move differently despite having the same amount of mass hanging on them. If there were three astronauts on three different springs, it would be impossible to compare their masses without knowing if the springs were the same. The students should conclude that for the spring device on the ISS to be useful, the astronauts need to know a specific value about the spring.

Tell the students they are going to make measurements of the springs to quantify this special difference between the different springs. Demonstrate that each of the three different springs will stretch to a different equilibrium length with the same amount of mass. Ask the students to focus on one spring, and choose two variables they can measure, one which they can change on purpose (independent variable) and the other which changes in response (dependent variable.)

Make sure students know how to read a metric ruler. Do this by passing out a printout of lines, and other shapes. Have students record their measurement in centimeters to the nearest millimeter of lines A, B, and C. Walk around the room and make sure their measurements are correct. It is possible that students will need to be told which part of the ruler has the metric measurements, and they may need a lesson on how to write their measurement in decimal form because many of them will be more familiar with using inches and fractions of inches. If only a few students need this assistance, deliver it to them individually or as a member of their group to assist them. Have a slide with visuals on cue in case there are more than a handful of students who need this mini-lesson.

The exact way students choose to measure their springs is up to them. If I were giving cookbook style instructions for this lab, I would instruct them to measure the entire length of the spring, not including the hook or eyelet at the top. But students might include the hook, or perhaps they will measure the height from the table to the bottom of the spring instead. These choices will make their data look different, but as long as they stay consistent in what they are measuring, and make a clear diagram showing exactly what they measured, the teacher will be able to assist them in interpreting their unique data. It is important though to make sure students understand that the measurement is made when the spring is not moving, as some will still be thinking of oscillating springs. Students sketch the lab set up and include notes in the sketch about from where to where they will measure the length of the spring each time the mass is changed.

Show students how to set up a data table. The column headers should identify exactly what is being measured, so this is something each group can decide. In my example, I have amount of mass as the independent variable, measured in grams, but some groups might use “number of 10g masses.” This also depends on the materials you have available.

I am particular about putting the unit of measurement in the column header and only values in each cell. Other teachers may have different styles. Tell students it is important to identify which spring their group is using because they will be comparing results later. Suggest that the first row of data can be for zero mass.

Data table for Spring #\_\_\_\_\_\_\_\_

|  |  |
| --- | --- |
| Amount of mass(g) | Measured length of spring (cm) |
| 0 |  |
|  |  |

Example of a student data table.

For the rest of the class period, allow students to work in groups to collect data. As homework, they should prepare a second data table and come to class prepared to collect data using a different spring.

As students collect data, do a quick check in with each group. You should have already measured all of the spring constants, so you can easily see if students are on the right track by looking at their data tables and doing a calculation with their data to see if they will have a similar spring constant.

Day 2

Start with more data collection. After enough time for students to have data sets for two different springs, hand out graph paper and guide students to setting up graphs for their data. The convention is to put the independent variable (mass) on the x-axis, and the dependent variable (length of spring) on the y-axis. Questions the students should consider when setting up their graphs:

* Do I need to include negative values on the x or the y-axis?
* How high should the numbers go on either axis?
* What will the increment of each grid line be?
* What should I title my graph?
* How should I label the axis?

In my experience, the Algebra I students have varying familiarity with how to make these decisions when setting up a graph. By having them work in groups, but each make their own graph, they can usually help each other. I circulate in the classroom and answer questions as needed for each group, but if there are many groups asking the same thing, I will call for the class’s attention and give a mini lesson on the board. The most common question I have to review as a class is how to decide what increment to number the gridlines in, so I like to have a slide prepared with visuals for explaining this. Since most students have made graphs only in math classes, rather than in science classes, they may also need to be told to include the units of measurement in the axis labels.

At this point, the graphs should just be plotted points, without a best fit line. If students are not done with their graphs at the end of class, their homework is to complete plotting the points.

Explain

To share results during the discussion in the following class, there are a few options. Students could sketch their graphs as a group on a whiteboard at the beginning of class, or students could scan and submit images of their graphs as homework and the teacher could compile examples of graphs into a presentation.

Day 3

At the start of day 3, display the students’ graphs at the front of the classroom. Ask them to take notes about what they notice, and any questions. Here are some key points the students may mention:

* All of the graphs have the points in a straight line
* We could add best fit lines to these graphs
* One group has a graph that goes down (negative slope), and I am wondering why it looks different from all the others that go up (positive slope). (This will happen if a group measures distance from table to bottom of spring instead of length of spring. Let students discuss and discover this; it is a great support for the necessity of clearly labeled axis.)
* Some of the graphs look steeper than others, and I notice that the two groups who used the really little spring have the steepest graphs. (It is important that which spring was used for each data set is included in the displayed data!)

After the class discussion, students return to their groups to answer the question “What are the ways the graphs for the same spring look the same? What are the ways the graphs for different springs look different? What does this tell us about how the graphs give information about the springs?”

Explore

Students add best fit lines to their graphs and write equations for the best fit lines. These are skills some students will already have, but have slides prepared to support the students in this process. With equations for the graphs, students can now compare with other groups. Find a group that made measurements using the same spring as you; are your equations similar? How do the equations for the two different springs your group measured the same/different?

Evaluate

Students answer analysis questions:

* What does the slope of a spring stretch vs mass graph represent?
	+ If my group used different variables on our graph, what is the slope of our graph representing?
* What does the y-intercept of the graph of spring stretch vs mass graph represent?
	+ If my group measured something besides spring stretch, what does the y-intercept represent?
* Use your graphs to predict how much your springs will stretch with 125g hanging on them.
* Explain if you could use your graph to predict how much your springs will stretch with 5000 g hanging on them. How about 4 g?
* Your graph started with discrete data points, then you added a continuous best fit line. Explain if you think spring stretch vs mass is a discrete or continuous function.

These questions can be posted as a google form, or other format which can be submitted through writing on the class internet site.

Elaborate

Follow up questions:

* All of these graphs were linear. We have the equations in slope intercept form. Can you write equations for other linear graphs given a data table of points? Given a y-intercept and a slope? Given two points? Given a best fit line shown on a coordinate plane?
* Given a linear graph, use it to answer questions about the value of each variable if you know one variable. Do the same if given only the linear equation.
* Distinguish between interpolating and extrapolating and be able to refer to the variables of a graph to explain what range or domain of values an extrapolation would be valid for.
* Use the variables and a data table to decide how to set up the axis of a graph: should negative values be included? What is the domain? What is the range? Is this continuous or discrete?

Day 4

Use problems from the textbook for more practice finding slope, writing linear equations, using other forms such as standard form and point-slope form.

Possible follow up activity:

The oscillation period of a spring can be measured for different masses. The graph of this data will have a $y\~\sqrt{x}$ shape and could be a nice introduction to non-linear data, without getting to involved in finding the equation. To do this, the students would need stop watches (which they have on their phones) and some instructions about how to time 5-10 oscillations then divide by the number of oscillations. Trying to time only one oscillation will not yield nice data.